Journal of Contemporary Issues in Business and Government Vol. 27, No. 1, 2021 P-ISSN: 2204-1990; E-ISSN: 1323-6903

https://cibg.org.au/

Portfolio optimization: An overview of integrated approaches and mathematical programing techniques

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***Abstract** Selection of stocks is a challenging task for investors and finance researchers because of the uncertainty of the return. In portfolio selection, the aim is to obtain a proper proportion of assets for getting maximum profit and least risk. The objective of his paper is to provide an overview of the present research in portfolio optimization with respect in mathematical programing techniques. For this purpose, 82 research papers appearing in the scholarly journal have been observed and investigated, and it has been concluding that fuzzy decision theory and goal programming establish the maximum number of mathematical programming techniques generated to solve the portfolio optimization problem.*

***Keywords.** Cluster analysis; analytical hierarchy process; optimization technique; portfolio optimization; forecasting.*
***MSC.** 65K10, 62P05, 94D05.*

1. INTRODUCTION

In portfolio preparation, the decision of stock selection depends on various constraints. To resolve the problem of portfolio selection, numerous models have been introduced such as, Markowitz model, sharp single index model, Konno and Yamazaki model, linear programming model etc. have been introduced.

The portfolio selection problem was initially introduced by Professor Harry Markowitz [1] and he was awarded with the Nobel Prize in Economics in 1990 for his great contribution to the portfolio selection problem. He introduced Markowitz model or mean-variance (MV) model in which return is calculated as the mean and risk as variance. He gave the concept that holding two or more assets are less risky than holding one asset, and this has become a foundation of modern portfolio theory. This model is conceptually sound in analyzing the return and risk of the portfolio. In Markowitz model, portfolio risk can be minimizing as follows:

Let us assume that r_i be the average expected return of the i^{th} stock, σ_{ij} is the covariance between return i and j, r_0 be the minimum return required by the investor and x_i is the proportion of the money which is invested in i^{th} asset.

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

Subject to $\sum_{j=1}^{n} x_j r_j \geq r_0$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_j \geq 0, j = 1, 2, 3, \dots, n.$$

If someone invests in 20 assets, then 190, that is n*(n -1)/2 covariance will have to be calculated, which, is the main problem with the Markowitz model. Due to these difficulties Sharpe introduced the Sharp Single Index Model [2], which is the simplified version of the MV model. The concept behind this model is that stocks vary mutually because of the common movement in the stock market and there is no effect beyond the market. In this regard x_i is assumed as the proportion of assets that is invested of the i^{th} asset, r_m is the return rate of the market index, α_i is the alpha-coefficient, β_i is the beta-coefficient, and ε_i represents the error, then the expected return of the portfolio is calculated as follows:

$$r_p = \sum_{i=1}^{n} x_i (\alpha_i + \beta_i r_m) + \varepsilon_i$$

The graphical representation of Sharpe model is shown in Figure 1.

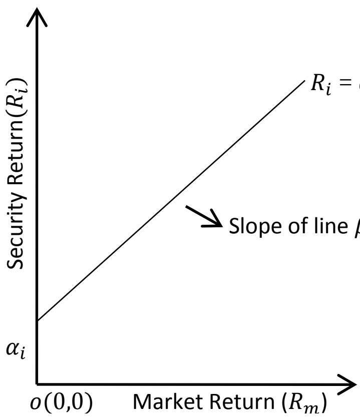

Figure 1 Graphical representation of Sharpe model

This further led to the development of the Capital Asset Pricing Model. Financial economist William Sharpe (Nobel Prize in economics) [3], introduced the Capital Asset Pricing Model (CAPM), in his book "Portfolio Theory and Capital Markets". The symmetrical expected return for risky assets was determined by the CAPM. This was related to the expected return and systematic risk of each asset or portfolio. CAPM gave the concept that each and every investment included two forms of risk, Systematic Risk and Unsystematic Risk.

Let us assume that r_{rf} is the return of the risk free asset, r_m is the expected return rate of the market and β_i is the sensitivity of the i^{th} asset then the return of i^{th} asset is calculated by following formula:

$$r_i = r_{rf} + \beta_i(r_m - r_{rf})$$

According to CAPM, all portfolio investments lie along the security market line in the beta return space. The security market line shown in figure 2.

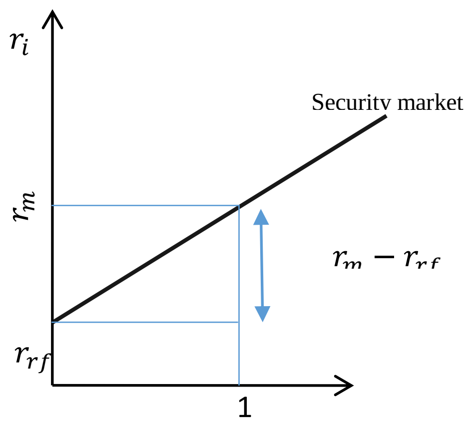

Figure 2 Security market line

The Black-Litterman model is a portfolio selection model that was developed by Black and Litterman [4] they solved the problem of unintuitive, input-sensitivity, highly-concentrated portfolios, and estimation error maximization.

If the number of views are k, the number of assets are n, τ is a scalar that lies between (0.025-0.05), Σ defines the covariance matrix of excess returns of order n, P is a (k×n) matrix with k views and n assets, Q is the view vector with order k×1, Ω is a diagonal covariance matrix of error terms from the expressed views of order k, and ∏ is the implied market return vector with order (n×1), then the new combined return vector E(r) with order (n×1) is formulated as:

$$E[R] = [(\tau\sum)^{-1} + P^{T}\Omega\, P]^{-1}\, [(\tau \sum)^{-1} \prod + P^{T}\Omega Q]$$

Konno and Yamazaki [5] introduced an improved and simplified version of Markowitz's Model both computationally and theoretically where risk is calculated as mean absolute deviation (MAD) instead of variance.

Assume that r_{jt} be the expected return of j^{th} asset in the period t, p_t defines the probability of period t, u_j is the upper limit then the portfolio risk can be minimized by the following formula:

$$min \sum_{t\epsilon T} p_t y_t$$

Subject to $y_t + \sum_{j\epsilon N}(r_{jt} - r_j)x_j \geq 0, t\epsilon T$

$$y_t - \sum_{j \epsilon N} (r_{jt} - r_j) x_j \geq 0, t \epsilon T$$

$$\sum_{j \epsilon N} x_j = 1$$

$$0 \leq x_j \leq u_j, j \epsilon N$$

$$y_t \geq 0, t \epsilon T$$

Speranza [6] presented a linear programming model related to portfolio selection and used semi absolute deviation to measure risk.
Assume that r_{jt} be the expected return of j^{th} asset in the period t, p_t defines the probability of period t, u_j is the upper limit then the portfolio risk can be minimized by the following formula:

$$min \sum_{t \epsilon T} p_t y_t$$

Subject to $y_t + \sum_{j \epsilon N} (r_{jt} - r_j) x_j \geq 0, t \epsilon T$

$$\sum_{j \epsilon N} x_j = 1$$

$$0 \leq x_j \leq u_j, j \epsilon N$$

$$y_t \geq 0, t \epsilon T$$

Many techniques have been used before applying the portfolio selection technique. These techniques are cluster analysis to categorize the stocks, the analytical hierarchy process for stock valuation, and for forecasting the future stock price. The previous study that was carried out is discussed in the next segments.

2. PORTFOLIO SELECTION AND OPTIMIZATION

Portfolio optimization is the procedure of picking the finest portfolio, out of all portfolios being considered with some objective. An overview of the portfolio selection and optimization is presented in this review. For this review, 82 research articles appearing in the scholarly journal have been scrutinized and investigated.

Before discussing the papers related to portfolio selection, we first discuss about some optimization techniques from the field of genetic algorithm, fuzzy-based, bio-inspired based etc. which are developed or updated.
Coello and Christiansen [7] presented a new multi-objective optimization technique based on min-max approach and applied genetic algorithm (GA) for solving two truss design multi-objective problem and compared the result with other mathematical programing technique. They showed that, GA gives better results and can be used as a trustworthy optimization tool.
Simon [8] introduced biogeography-based optimization algorithm for optimization problems. He discovered the mathematics of natural biogeography and discussed how to use it to solve the optimization problems. It is based on the concept of migration and mutation. It is population based technique.

Mavrotus [9] proposed an epsilon-constraints method to solve multi-objective optimization problems for both continuous and discrete variables. This method has been already applied using general algebraic modelling language software.

Teaching-learning-based optimization developed by Rao *et. al.*, [10] which is built on class-room concept and the working method is separated into two phases such as teacher phase and learner phase. This method is introduced for mechanical design problem but can be extended to other optimization problems. Rao and Patel [11] developed the elitism idea in TLBO algorithm and investigated the performance and finally compared it with other optimization techniques.

Cheng [12] addressed a new method to solve FMOLP and applied weighted max-min method instead of weighted adaptive approach. Fuzzy objective or constraints transform into crisp linear programming problem by using deviation degree and all the coefficients are triangular fuzzy number and constraints are fuzzy equality and inequality also given the numerical example for the proposed method.

Bandyopadhyay and Saha [13] presented a detailed description of various metaheuristics optimizing techniques namely genetic algorithm and simulated annealing for solving both single objective and multi-objective optimization problem.

Bharati and Singh [14] presented a comparative study of intuitionistic fuzzy optimization and fuzzy optimization for solving multi-objective linear optimization problem. This study also involved optimization using linear and non-linear with membership and non-membership functions and concluded that non-linear membership and non-membership functions gives better results.

Dubey and Mehra [15] introduced a different way to design fuzzy multi-objective linear programming problem in bipolar viewpoint which permits to differentiate between negative and positive preferences.

Pareto optimality solution concept for multi-objective fuzzy linear programming was given by Dubey and Mehra [16] who stretched this study to multi-objective linear programming problem with interval vagueness including hard and flexible constraints.

Out of these optimization techniques fuzzy based optimization is one of the most famous technique which has been applied for portfolio selection.

Chiam *et. al.*, [17] have projected an order-based approach for an evolutionary multi-objective portfolio selection problem that can be extended to manage floor and ceiling constraints and cardinality constraints simultaneously. This approach generates a better image for an efficient frontier as compared to other traditional representations. Branke *et. al.*, [18] has projected an envelope-based multi-objective evolutionary algorithm for portfolio selection, which is a mixture of a multi-objective algorithm with an embedded algorithm for parametric quadratic programming. In this proposed method, the evolutionary algorithm creates a bunch of convex subsets of the search space. An efficient frontier is generated by these subsets, called envelopes.

Lin *et. al.*, [19] proposed three models for portfolio selection, with very little transaction cost. The proposed model involved a modified form of the Markowitz' model, using a fuzzy multi-objective decision-making approach and a genetic algorithm, which helped minimize the gap between the target and the evaluated portfolio. The results very closely represent the efficient frontier and model, which is based on fuzzy multi-objective decision-making, which is particularly suggested for portfolio selection, due to its flexibility. Jana *et. al.*, [20] presented a multi-objective nonlinear programming approach with transaction cost, assuming risk, return, liquidity and entropy as objective functions. The proposed model is solved by the fuzzy nonlinear programming technique. Hasuike and Katagiri *et. al.* [21] considered the fuzzy programming problem for portfolio selection and applied the sensitivity analysis for changing the investors' subjectivity. Bhattacharya *et. al.*, [22] proposed a new framework of a fuzzy MVS portfolio selection, based on the concept of interval valued numbers. Three models have been designed based on future financial markets and solved by using the hybrid intelligence algorithm. Sekaran and Ramaswami [23] presented a portfolio optimization model by applying the neuro-fuzzy framework and used the Sensex index as a benchmark for testing the efficiency of the forecasted and optimized results. They concluded that the neuro-fuzzy framework gives a more accurate result, as compared to other optimization techniques. A fuzzy method was introduced by Sanokolaei *et al.* [24] for portfolio optimization, on the basis of six different criteria of values at risk function, which concludes that the mean absolute deviation function model is the best out of these six models, and this comparative study is done using the Kupiec failure probability number. Two possibilistic mean–semi variance models, with real constraints, were given by Liu and Zhang [25] and the fuzzy multiobjective programming approach was used to solve this model, in which, return, risk, liquidity, and liquidity risk were assumed to be fuzzy variables and calculated by using possibilistic mean and possibilistic semivariance. Fuzzy linear programming was presented by Konak and Bagci [26], for portfolio optimization. They applied Warner's model as a base for portfolio optimization, which analyzed and examined the past performance of stocks. The resultant portfolio created by the proposed model was expected to give a return rate of 0.7% with 9.6% risk. A new risk index variable, which was called the equilibrium risk value (ERV) of the random fuzzy expected value (EV), was addressed by Wang *et. al.* [27], where he applied the EV-ERV model for portfolio selection. The efficiency of the proposed model was compared by the traditional stochastic optimization technique. Mehlawat and Gupta [28] developed a portfolio selection model in view of the fuzzy chance constrained multi-objective programming, with the aim of getting maximum returns and liquidity. The numerical section of this article involved the application in real-world and the efficiency of the proposed model. Mehlawat and Gupta [29] addressed a portfolio selection approach in a fuzzy environment and applied a fuzzy mathematical programming model based on hybrid bi-objective credibility and considered three criteria, risk, return, and liquidity simultaneously.

Cura [30] presented a heuristic approach for portfolio selection and applied the particle swarm optimization (PSO) algorithm on a data set of five indices from five different countries and used the cardinality constraints MV model. The results were compared with the tabu search, and genetic algorithms, with simulated annealing, and it was concluded that PSO was effective in solving the portfolio optimization problem. Liu *et. al.*, [31] developed a proportion entropy for computing the diversification degree of a portfolio, by converting it into crisp nonlinear programming, using the fuzzy decision-making theory and multi-objective programming. For solving this model an upgraded PSO approach was applied. Zaheer [32] used the Shanghai Stock Exchange data to develop a hybrid PSO technique for portfolio optimization. He considered two different models with short sale and without short sale.

Anagnostopoulos and Mamanis [33] investigated the ability of the non-dominated sorting genetic algorithm II (NSGA-II), Pareto Envelope-Based Selection Algorithm (PESA), and strength Pareto-Evolutionary Algorithm 2 (SPEA2) for solving a complex bi-objective portfolio selection problem. They concluded from the results that the proposed algorithm was effective and trustworthy, and the performance was not dependent on the risk function. An integrated approach was introduced by Gupta *et. al.*, [34] for portfolio selection, in which, the financial assets were classified by a support vector machine and a real-coded genetic algorithm was applied to solve the mathematical model. Assets were classified in three groups on the basis of risk, return, and liquidity. Gupta *et. al.*, [35] presented a multi-objective credibilistic model for a portfolio rebalancing model with fuzzy chance constraints and developed a hybrid intelligent algorithm, integrating fuzzy simulation and a real-coded genetic algorithm for calculating the proposed mathematical model. Mittal and Mehlawat [36] presented a multi-objective model for the portfolio rebalancing problem and solved the proposed model by developing a real-coded genetic algorithm. They also considered a financial market state that the portfolio could be altered to after a fixed time-period, by trading the assets. Hadi *et. al.*, [37] applied a pareto based enhanced genetic algorithm with four further constraints for portfolio optimization and used data selected from the Egyptian Exchange. They show that the proposed model is best among all the conventional optimization models. Mashayekhi and Omrani [38] applied the second version of a non-dominated sorting genetic algorithm (NSGA-II) for portfolio optimization, using data taken from Tehran Stock Exchange. Chen [39] presented an uncertain mean-variance-skewness portfolio selection model with the criteria transaction costs, bounds on holdings, cardinality of the portfolio, and minimum transaction lot constraints, and developed a hybrid approach firefly algorithm-genetic algorithm (FA-GA) for solving the proposed model.

Miyahara and Tsujii [40] studied the portfolio optimization problem in the case of the Levy process model and applied the “risk sensitive value measure method” for portfolio optimization, and for financial risk assets assumed the risk-sensitive value measure.

Stoyan and Kwon [41] addressed a complex stochastic goal mixed-integer programming model for stock and bond portfolio. Masmoudi and Abdelaziz [42] presented a bi-objective stochastic programming portfolio optimization model, which was solved by goal programming with the objectives of return and risk. Ghahtarani and Najati [43] presented a robust optimization goal programming for the portfolio selection problem. Gupta *et. al.*, [44] developed a credibilistic model for portfolio selection and used goal programming and real-coded genetic algorithm to solve the model and also involve the application of portfolio selection in the real-world. Gupta *et. al.*, [45] employed the multi-criteria credibilistic structure for the portfolio rebalancing problem. Goal programming and hybrid intelligent algorithm are applied for solving the proposed model in which the hybrid intelligent algorithm is solved using fuzzy simulation and real-coded genetic algorithm. Lam and Lam [46] applied a goal programming model using mean return and tracking errors for optimizing the portfolio. Tamiz and Azmi [47] proposed the extended factors of stocks and applied goal programming for portfolio selection. They applied three alternatives of goal programming, namely weighted programming, lexicographic programming, and minimax programming.

Huang and Qiao [48] proposed a model for solving multi-period portfolio selection in which experts evaluated the security returns and proposed an uncertain risk index adjustment model.

This model efficiently solves the multi-period portfolio selection model. Sadjadi *et. al.*, [49] presented a fuzzy multi-period portfolio selection model and discussed the uncertainty of the lending and borrowing rates. These rates are in the form of a fuzzy triangular number. Mehlawat [50] presented a new multi-criteria credibilistic multi-period portfolio selection approach including liquidity, cardinality, and diversification constraints. Risk and portfolio risk is measured by credibilistic entropy. Saglam and Benson [51] presented the multi-period portfolio optimization problem in a mean-variance framework including diversification-by-sector constraints, buy-in-thresholds, transaction costs, and conditional value-at-risk.

Zhang *et. al.*, [52] introduced two credibilistic mean-variance portfolio adjusting models with fuzzy returns, and for the adjustment process, lending, borrowing, transaction cost, additional risk assets, and capital are used.

Hitaj and Mercuri [53] investigated the effect of higher movement in portfolio selection when parametric and non-parametric models were used.

Liu and Qin [54] introduced the idea of semi-absolute deviation for uncertain variables and recognized the mean semi-absolute model for this. This proposed optimization model was effective and important for uncertain portfolio problem.

Bruni *et. al.* [55] advocated a linear bi-objective optimization to enhanced indexation (EI), which maximized return and minimized risk in the learning period. Goel *et. al.* [56] created portfolios for the problems of index tracking and enhanced indexing where he applied mixed conditional values at risk for these portfolios. Furthermore, they presented a two-way process for EI problems such as a discrete Markov chain model for filtration assets, and allocated optimal weights to filtered assets.

A portfolio selection model, based on possibility theory with a parameter fuzzy random variable, proposed by Sadati and Doniavi [57] was solved using a harmony search algorithm. They showed that the proposed method efficiently solved the portfolio selection problem. Liu *et. al.* [58] employed a possibilistic international asset allocation model and developed a novel time-variant differential evolution, with a harmony search algorithm for the solution.

Rahnamay *et. al.* [59] applied the robust optimization technique for portfolio selection and compared risk and return of the portfolio with the classic model.

Bacanin and Tabu [60] employed a modified firefly algorithm (mFA) with entropy constraints for the cardinality constrained mean-variance portfolio optimization problems, and showed that the proposed mFA was better when compared to other algorithms.

Sharma and Mehra [61] introduced sectoral portfolio optimization which is based on financial analysis. They optimized stocks in each sector on the basis of financial analysis and then created an optimal portfolio with different weights.

Qu *et. al.* [62] presented an efficient model for solving portfolio problems, which was large-scale optimization problems, by introducing two asset preselection processes. He applied MOEA/D, MODE-SS, MODE-NDS, MOCLPSO, and NSGAII multiobjective evolutionary algorithms to check the efficiency of the proposed model.

Chen *et. al.* [63] evaluated fuzzy portfolio efficiency in different risk procedures namely possibilistic variance, possibilistic semivariance, and possibilistic semi-absolute deviation, and compared this model with a real frontier approach.

Sharma *et. al.* [64] applied the Omega ratio to regulator downside risk by using distribution-dependent thresholds for portfolio optimization. Sharma *et. al.* [65] introduced under, and over-achievement variables in second order stochastic dominance (SSD) and proposed a linear optimization model for maximizing mean returns by creating constraints using relaxed SSD. In [64], the omega ratio was applied for optimization and the advanced version of the Omega ratio optimization model was introduced by Sharma and Mehra [66], which involved the Omega ratio optimization model and mean-risk model.

Javid and Tafti [67] compares entropy value at risk (EVaR) and conditional value at risk (CVaR) for sample based portfolio optimization problem and concluded that EVaR gives better results as compare to CAaR.

Zhang *et. al.*, [68] introduced a simulation-and-regression method to optimize dynamic portfolio problems with the parameters such as transaction costs, liquidity costs and market impact.

Rahmani and Khelil [69] introduced a new technique to solve portfolio problems. This is a two-way technique namely principal component analysis (PCA) and genetic algorithm. PCA organize the activities into classes and then optimization is done by mean absolute deviation with genetic algorithm.

Li and Zhang [70] investigated the portfolio optimization model with higher order moments and assume kurtosis as an objective function and the variance, transaction costs, skewness and mean as the constraints. They conclude that the transaction costs are the important factor for portfolio optimization model and study the different form of correlation between kurtosis and variance. The non-convexity and higher order moment are successfully avoided by this method.

Review in tabular form according to their common approaches are given in the following table.

Table 1. Research paper and the optimization technique.

Sr. No.	**Year**	**Author's Name**	**Approaches**	**Optimization Technique**
1	2008	Chiam et. al., [17]	Evolutionary algorithm	
2	2009	Branke et. al., [18]		
3	2008	Lin et. al., [19]	Fuzzy Decision Theory	FMODM
4	2009	Jana et. al., [20]		FMONLP
5	2010	Hasuike and Katagiri et. al. [21]		FPP
6	2012	Sekaran and Ramaswami [22]		neuro fuzzy framework
7	2011	Bhattacharya et. al., [23]		fuzzy MVS, hybrid intelligence algorithm
8	2014	Sanokolaei et al. [24]		fuzzy technique
9	20	Liu and Zhang [25]		FMOP

	2013			
10	2016	Konak and Bagci [26]		FLP, Warner's model
11	2016	Wang et. al. [27]		EV-ERV model
12	2014	Mehlawat and Gupta [28]		fuzzy chance constraints MOP
13	2014	Mehlawat and Gupta [29]		FMP
14	2009	Cura [30]	Particle Swarm Optimization	PSO
15	2013	Liu et. al., [31]		FDM
16	2018	Zaheer [32]		hybrid PSO
17	2011	Anagnostopoulos and Mamanis [33]	Genetic Algorithm	NSGA, PESA, SPEA
18	2012	Gupta et. al., [34]		SVM
19	2013	Gupta et. al., [35]		
20	2014	Mittal and Mehlawat [36]		
21	2016	Hadi et. al., [37]		pareto based GA
22	2016	Mashayekhi and Omrani [38]		NSGA
23	2018	Chen [39]		FA-GA
24	2011	Miyahara and Tsujii [40]		Risk sensitive value measure model
25	2011	Stoyan and Kwon [41]	Goal Programming	
26	2012	Masmoudi and Abdelaziz [42]		
27	2013	Ghahtarani and Najati [43]		
28	2013	Gupta et. al., [44]		GA
29	2013	Gupta et. al., [45]		GA, fuzzy simulation
30	2016	Lam and Lam [46]		

31	2017	Tamiz and Azmi [47]		
32	2012	Huang and Qiao [48]	Multi-period model	
33	2011	Sadjadi et. al., [49]		fuzzy based
34	2016	Mehlawat [50]		entropy based
35	2018	Saglam and Benson [51]		mean-variance framework
36	2011	Zhang et. al., [52]		mean-variance adjusting model
37	2013	Hitaj and Mercuri [53]		parametric & non-parametric model
38	2012	Liu and Qin [54]		mean semi absolute model
39	2015	Bruni et. al. [55]	Enhanced indexing	
40	2018	Goel et. al. [56]		
41	2014	Sadati and Doniavi [57]	Harmony search algorithm	
42	2018	Liu et. al. [58]		
43	2015	Rahnamay et. al. [59]	robust optimization technique	
44	2014	Bacanin and Tabu [60]	modified firefly algorithm	
45	2017	Sharma and Mehra [61]	sector based optimization	
46	2017	Qu et. al. [62]	Evolutionary algorithm	
47	2018	Chen et. al. [63]	variance based technique	
48	2017	Sharma et. al. [64]	Omega ratio	
49	2017	Sharma et. al. [65]	SSD	
50	2017	Sharma et. al. [66]	Advanced form of omega ratio	
51	2019	Javid and Tafti [67]		compare EVaR and CVaR
52	20	Zhang et. al., [68]		simulation-and-regression

	19			method
53	20 19	Rahmani and Khelil [69]	GA	PCA and GA
54	20 19	Li and Zhang [70]		higher order moment portfolio problem

3. CLUSTER

Cluster analysis is a process of grouping similar objects in the same cluster, which are different from other cluster's objects. A different investor has a different approach toward selecting stocks. Generally, they are focused only on return, risk, and liquidity, hence, stocks are divided into three clusters, namely, high return stocks, less risky stocks, and liquid stocks according to the investors' choice.

Dose and Cincotti [71] developed the stochastic optimization technique for the index tracking problem and applied the hierarchal clustering technique to group the assets of the S&P 500 index into two different groups. Nanda *et. al.*, [72] developed a comparative study in between the k-means clustering, fuzzy c-means clustering, and self-organizing maps (SOM) for stock selection and portfolio optimization, done in the Markowitz model. They analyzed k-means clustering and created the densest clusters, when compared with fuzzy c-means and SOM.

Gupta *et. al.* [73] presented a hybrid approach for portfolio selection that combined multiple methodologies namely behavioral surveys, cluster analysis, analytical hierarchy process (AHP), and fuzzy mathematical programming. Gupta *et al.* [74] applied the AHP for calculating suitability of assets according to investors' interest and hybrid approach for portfolio selection. In this article, absolute deviation and semi-absolute deviation functions were applied for measuring risk. Gupta *et. al.*, [75] used the AHP technique to achieve the ethical performance of stocks and MCDM (FMCDM) technique for portfolio selection. Long *et. al.*, [76] proposed a model for portfolio optimization, which included the multi-objective genetic algorithm and fuzzy C-means clustering, to categorize stock data into several clusters, based on return rate and risk. Lemieux *et. al.*, [77] presented the comparative study of K-means, K-medoids, and hierarchical clustering techniques to determine the effects of the riskiness of different portfolios.

4. ANALYTICAL HIERARCHY PROCESS

Thomas L. Saaty addressed AHP, which is a multi-criteria decision making (MCDM) tool in the 1970s [78]. AHP is a very important tool where many alternative needs are to be evaluated. AHP is used for evaluation of assets as per investor's preference. Ranking of assets can be done with the help of AHP.

AHP has several applications in various fields, such as, medical, manufacturing area, industry, government, education, personal, management, engineering, social, and the like.

Ho [79] presented a review of applications of integrated AHP from 1997 to 2006. They investigated which application field was primarily applied in AHP, what type of integrated

AHP approaches were mostly applied, and which journals were published that integrated the AHP approaches.

Ishizaka and Labib [80] presented a review of the 7+ methodological developments of AHP, instead of its application in various fields. They developed a complete review of the flexibility and drawbacks of modeling, comparisons, consistency, ranking scale, weight analysis, and sensitivity analysis.

Subramanian and Ramanathan [81] presented a review of AHP in the field of operation management. This literature listed 291 journal research articles from 1990 to 2009, which research the applications and gaps of AHP in operation management.

Ho and Ma [82] presented a review of the integrated AHP approaches from 2006 to 2017 based on the study of 88 journal articles. This article is a continuation study to Ho [79]. They investigated which application field had primarily applied AHP, which type of integrated AHP approach was mostly applied, which journals were published that integrated AHP approaches.

Oyatoye *et. al.*, [83] applied AHP to decide the importance of the variety of criteria and alternatives. Heidari and Soleimani [84] used AHP and the Markowitz model for portfolio selection. Mehlawat [85] presented a detailed computation procedure of AHP and determined the suitability performance score of the assets with the help of the AHP model. He applied the FMCDM technique to obtain optimal portfolios. Solimanpur *et. al.*, [86] presented a multi-objective genetic algorithm and AHP with three-level hierarchies for portfolio optimization.

5. FORECASTING

Stock price prediction is an important and interesting topic for investors. Accurate prediction is not possible for stock prices due to uncertainty of the market; it is the only probable value that is calculated based on past performance of the stock. No one is assured about the ups and downs of the market. Simulation is based on probabilities, not certainty. Exact forecasting of stock price is a challenging and complex task, as we do not have any information about future observation or market strategy. Its distribution and variations are determined on the basis of historical data. There are a lot of techniques to forecast the future stock price, such as the ARIMA model, Monte-Carlo simulation, artificial neural network, regression model, etc. The ARIMA model is the most common technique for forecasting future stock prices, but the Monte-Carlo simulation also forecasts efficient results. This technique is designed on a random value. This method gives thousands of results, and each result has a different random value. In this research, forecasting is done by Monte-Carlo simulation, which is based on random value and gives thousands of results with corresponding random number.

Adebiyi *et. al.*, [87] presented a comparative study for forecasting the accuracy of the ARIMA model and artificial neural network. Mondal *et. al.*, [88] applied the ARIMA model for predicting future stock prices from different sectors and calculated the accuracy. Jadhav *et. al.*, [89] proposed a hybrid approach, which combine artificial neural networks (ANNs) and the ARIMA model, to give more efficient, forecasted results, as compared to ANNs. Alrabadi and Aljarayesh [90] observed that Monte Carlo was the most accurate forecasting technique as compared to simple moving average and exponential moving average techniques. Sonono and Mashele [91] provided a relative analysis of continuous time models General Brownian Motion and Variance Gamma to predict the direction and accuracy of the stock prices. They used the Monte Carlo methods Quasi Monte-Carlo and Least-Square Monte-Carlo. Zhou et. al. [92] presented two entropy-based optimization models for portfolio optimization and applied the fuzzy time series technique for forecasting.

Table 2, represent the integrated approaches along with the optimization technique for portfolio selection problem.

Table 2. Portfolio optimization and integrated approaches.

Sr. No.	**Year**	**Author's Name**	**Integrated Approach**	**Optimization Technique**
1	2005	Dose and Cincotti [71]	Hierarchy clustering	Stocastic optimization
2	2010	Nanda et. al., [72]	K-means & c-means clustring	Markowitz model
3	2010	Gupta et. al. [73]	Investor behavior, k-means clustering, AHP	Fuzzy mathematical programing
4	2011	Gupta et al. [74]		Konno & Sprenza model
5	2013	Gupta et. al., [75]		FMCDM
6	2014	Long et. al., [76]	C-means clustering	GA
7	2014	Lemieux et. al., [77]	k-means, hierarchical clustering	
8	2010	Oyatoye et. al., [83]	AHP	
9	2013	Heidari and Soleimani [84]		Markowitz model
10	2016	Mehlawat [85]		FMCDM
11	2015	Solimanpur et. al., [86]		GA
12	2014	Adebiyi et. al., [87]	ARIMA, ANN	
13	2014	Mondal et. al., [88]	ARIMA	
14	2015	Jadhav et. al., [89]	ARIMA, ANN	
15	2015	Alrabadi and Aljarayesh [90]	Monte-Carlo	
16	201	Sonono and Mashele	QMC, LSMC	

	5	[91]		
17	2015	Zhou et. al. [92]	Fuzzy time series forecasting	fuzzy entropy based optimization

6. RESEARCH GAP

We observe from the literature review that k-means and fuzzy c-means algorithm are used for clustering. K-means calculates only the Euclidean distance and the initial selection of the centroid is random; at the same time c-means also measure the distance from the centroid which is a common drawback in both.

Hierarchy of AHP is focused only on the criteria of return, risk, liquidity, alpha-coefficient, beta-coefficient and value at risk. There are some more important and new factors which are not considered for valuation of stocks.

Post-optimality test has not been considered much for portfolio optimization, although it is an improvement for real solutions.

7. CONCLUSION

This article represents a literature review on portfolio selection and optimization. For this purpose, 82 national and international research articles from scholarly journals have been observed and examined. Hopefully, this review will give a clear overview of portfolio selection, and researchers will gain detailed information related to portfolio selection.

From this review we conclude that fuzzy decision theory is the most common optimization technique also genetic algorithm and goal programming having lots of attention for optimization.

Generally, the methodologies involved in portfolio selection are, investor behavioural survey, cluster analysis, the analytical hierarchy process, the optimization technique, and the ARIMA model; yet, there are many opportunities in this research for further improvement in portfolio selection.

There has been a considerable amount of research related to investments, but still there is a great possibility for new and innovative ideas in this area.

There are many more opportunities in the field of investor behavior topology in respect stock picking ability. Stock selection on the basis of present return rate and risk is a major challenge for researchers. Stock valuation under some new criteria other than risk and return is an important task also. Yet the selection of objective functions other than risk and return is the search topic.

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