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ECONOMIC CYCLES AND THE THERMODYNAMIC UNCERTAINTY RELATIONS

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ABSTRACT
In the century and a half since Maxwell first conjured his "finite being" which Lord Kelvin subsequently dubbed a "daemon", researchers have explored the connections between non-equilibrium thermodynamics, entropy, and information theory. In recent years various Thermodynamic Uncertainty Relations (TURs) have been derived to inform upon the relationship between the entropy production and the precision possible in thermodynamic machines and processes. In this paper the recently derived TURs are applied to a hypothetical thermodynamic economy. The TURs define the lower bound on the total entropy production of the economy. Changes in the economy's entropy production rate have important consequences for the stability of the economic systems, the growth of inflation and play a central role in the evolution of the business cycle. This new perspective has important implications for policy makers, researchers, and other economic actors.

KEYWORDS: Entropy; Landauer's Principle; Multiscale Entropic Lifecycle; Thermodynamic Uncertainty Relations; TURs; Entropic Yield Curve; Business Cycle.

INTRODUCTION
Researchers have explored the connections between economics and thermodynamics and have produced numerous models detailing these connections. For example, (Chen, 2005) attempted to describe the physical foundations of economic activity in terms of thermodynamics, while (Kümmel, 2011) related thermodynamic energy conversion and entropy production to natural, technological, and social evolution. Other researchers such as (Baumgarter, 2002) focused on the thermodynamics of waste from economic activities, while (Grisolia et al., 2020) introduced a new bioeconomic indicator based on the exergy analysis of dissipation and irreversibility.

Additionally in recent years many new thermodynamic theories have been developed. These include the derivations of the thermodynamic uncertainty relations (TURs) (Barato and Seifert, 2015). Researchers have used these new tools to study a wide variety of classical, quantum, chemical, mechanical, and biological systems. In this paper TURs will be used to provide another perspective on the behavior of interest rates and inflation over the business cycle.

MATERIALS AND METHODS
THERMODYNAMIC ECONOMY
The thermodynamic economy modeled in this paper is an open nonequilibrium system that exchanges matter, energy, and information with the external environment extending the model developed by (Parker, 2017, 2018, 2019, 2024). Our economy is not a single monolithic structure but is composed of many interlinked subsystems. The various parts of the open economy have different proportions of the matter and energy needed as inputs[i]. For example, stock markets have very little direct matter exchanges with the outside environment. Instead, energy (and the information embedded within it) drive the market. Construction companies on the other hand may have a larger relative proportion matter to energy as required inputs. However, both systems have information as an important input in addition to the other needed inputs.

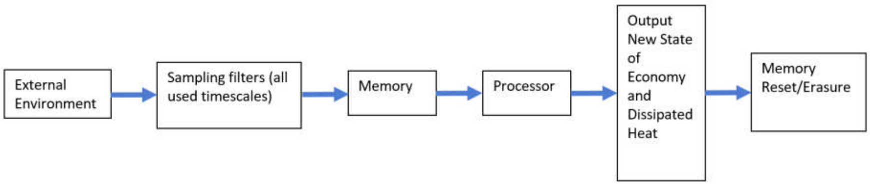


Figure 1: Thermodynamic Economy

Figure 1 presents a depiction of the fundamental parts of the thermodynamic economy. These structures include the external environment from which the economy receives matter, energy, and information. The decentralized thermodynamic economy is composed of several interacting parts as seen in Figure 1. In reality the many interacting parts and processes complete their steps at different times and in parallel. This simplified hypothesized economy highlights the components central to the analysis while remaining consistent with the rules of thermodynamics.

A wide range of information is sampled from the environment at various timescales in a process represented by filters. This new information from the outside environment is stored in the economy's memory. The processor takes the information and uses it to compute the next state of the economy. The economy is reorganized in the next step to the state previously computed, and waste heat is expelled into the surrounding environment. The final step following from the Landauer's Principle is the erasure of the memory to prepare for the arrival of new information from the environment. (Landauer, 1961) computed the theoretical minimum energy required to erase one bit as $E \geq k_b T ln2$. This erasure leads to the irreversible loss of information as the entropy outside of the economy is increased and the net entropic accounting is balanced.

THE ENTROPIC YIELD CURVE

The entropic yield curve was derived in (Parker, 2017) and provides a relationship between an economy's ability to process information and interest rates. Specifically, it was demonstrated that the entropic yield curve also represents the amount of entropy generated at each timescale:

$$r = \left(1 - e^{-C_1\left(1-\frac{R}{C}\right)}\right)\frac{ln\left(\sqrt{t}\right)}{t} - \left(e^{-C_1\left(1-\frac{R}{C}\right)}\right)\frac{\ln(\sigma)}{t} = Total\ Entropy\ (Parker, 2017)$$

where

$$r_t = the\ interest\ rate\ at\ t\ maturity$$

$$\frac{R}{C} = \frac{Economic\ or\ market\ information}{Capacity\ to\ process\ information}$$

$$\sigma = error\ in\ computation\ or\ communication$$

$$C_1 = input\ parameter\ determining\ the\ distinguishablility\ of\ the\ inputs$$

This analysis is extended in this paper further developing the relationship between the economy's entropy production and thermodynamic and computational limits.

LANDAUER'S PRINCIPLE AND THE THERMODYNAMIC ECONOMY

In this paper's model the economy is represented as a large and decentralized information processing computer. This hypothetical computer gets information inputs from the economy and uses these inputs to compute the next state of the economy. In other words, information is being used to organize the real structure of the economy. This model can be viewed as an example of Maxwell's famous demon who hypothetically decreases the entropy of a system of interest using only information. By the Second Law of Thermodynamics this entropy decrease must be accompanied by an entropy increase elsewhere of at least the same magnitude.

The hypothetical economic computer determines the future state of the economy using the inputs and its internal programming. After this calculation the economy's computational system must erase the no longer needed data filling its finite computer memory. (Landauer, 1961) computed the theoretical minimum energy required to erase one bit as $E \geq$

$k_b T ln2$. This erasure leads to the irreversible loss of information as the entropy outside of the economy is increased and the net entropic accounting is balanced.

While modern computers operate at orders of magnitude above the Landauer energy minimum, they are still bound by another consequence of the theory. For a given level of technology there is a maximum rate at which heat can be dissipated as information is erased. This means there exists a maximum computation rate for any finite technology. Exceeding this rate leads to incomplete erasures, calculation mistakes, and even damage to the computational system. The less efficient the economy's information processing capability, the lower its currency's purchasing power and the higher its inflation rate.

(Parker, 2017, 2018, 2019, 2024) used Landauer's Principle and the entropic yield curve to explain the ending of the business cycle. Specifically, it was found that at the shortest timescales the economy exceeds the (Shannon, 1949) and Landauer limits. In the next sections the entropic yield curve is used to model the evolution of the yield curve over the business cycle from the perspective of the thermodynamic uncertainty relations.

THERMODYNAMIC ECONOMY AND THE MULTISCALE ENTROPIC YIELD CURVE

Using the entropic yield curve, (Parker, 2017, 2018, 2019, 2024) examined the business cycle from the perspective of the multiscale entropic lifecycle (MSELC) of the yield curve. Specifically, the structure and evolution of the yield curve is described as a process dominated by the shortest end of the yield curve. Late in the business cycle the total volume of data in the shorter timescales exceeds the economic system's channel capacity or the Landauer's limit as seen in (Parker, 2017, 2018, 2019, 2024).

Figure 2 displays the typical sequence of yield curve shapes seen throughout the business cycle. The entropic yield curve easily generates these shapes as described in (Parker, 2017, 2018, 2019, 2024). Most of the movement in the curve occurs in the shorter end of the maturity axis.

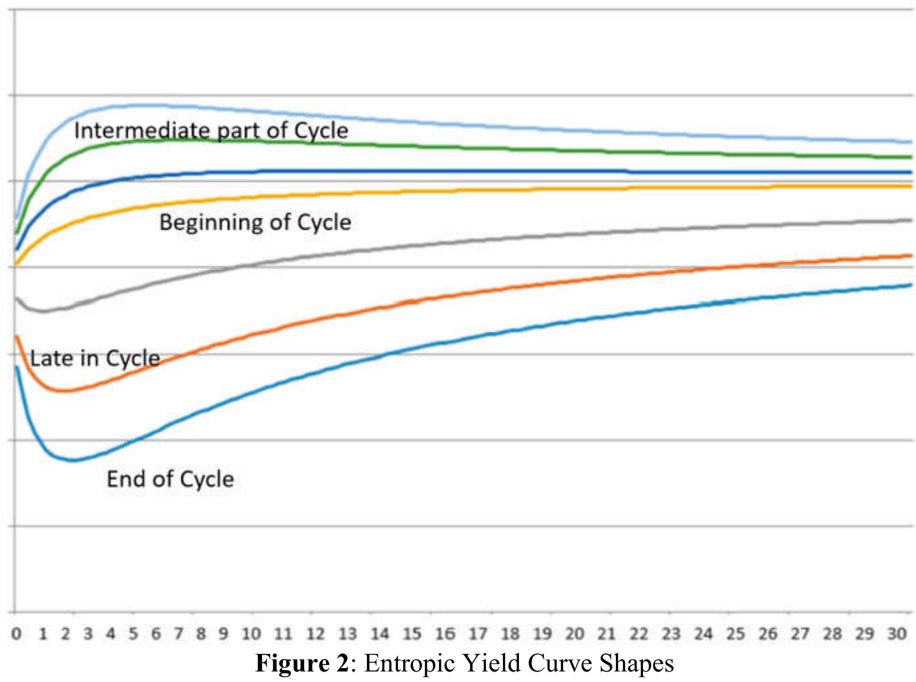


Figure 2: Entropic Yield Curve Shapes

At the end of the business cycles the yield curve takes the shape shown below in Figure 3. This shape implies that at the lowest end of the time to maturity(timescale) axis entropy production falls below that of the longer term to maturity (Parker, 2017, 2018, 2019, 2024). It is notable that this terminal shape seen at the end of all business cycles is not described elsewhere in the literature. This omission may be due to the lack of traditional explanations for this specific shape in standard economic theory.

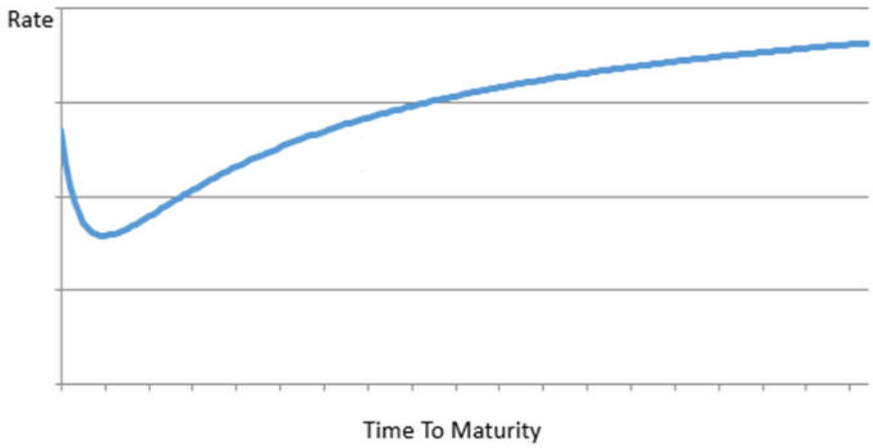


Figure 3: Yield Curve End of Business Cycle

THE YIELD CURVE AS A MULTISCALE ENTROPY MEASURE

(Parker, 2017) provided a new derivation of the yield curve based on information theory. Under the assumption that there are limits on the communication of information through time and space the author demonstrated that the yield curve is in fact a measurement of the entropy at various time scales. Specifically (Parker, 2017) derived the following equation:

$$Total\ Entropy = rt = (1-p)ln(\sqrt{t}) - (p)ln(\sigma)$$

$$Growth\ Rate\ of\ Entropy\ at\ time\ scale\ t = r_t = (1-p)\frac{ln(\sqrt{t})}{t} - (p)\frac{\ln(\sigma)}{t}$$

Final form introduced in Parker 2017

$$r_t = \left(1 - e^{-C_1\left(1-\frac{R}{C}\right)}\right)\frac{ln(\sqrt{t})}{t} - \left(e^{-C_1\left(1-\frac{R}{C}\right)}\right)\frac{\ln(\sigma)}{t}$$

Below is presented an example plot of the entropic yield curve for various values of R/C. All shapes of the yield curve can be modeled by this formula by the manipulation of variable R/C. The variable R/C represents a measure of the loss of information in the economy as explained in (Parker, 2017). As the economy evolves through a business cycle the yield curve moves through these shapes and typically ends the business cycle with the last “check mark” or “terminal curve” shape seen at the bottom of **Figure 4** below.

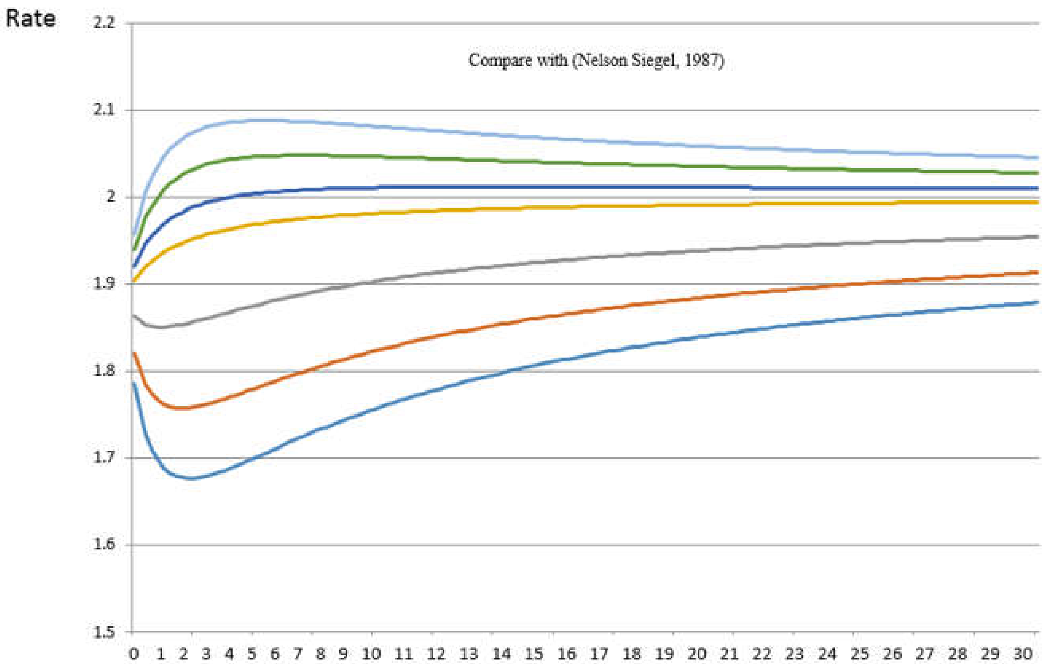


Figure 4: Entropic Yield Curve

From the above equations and in (Parker, 2017, 2018, 2019, 2024) we can now see the yield curve as a measure of entropy or loss of information at each maturity or timescale. We can examine the multiscale entropy of the economy by looking directly at the yield curve in effect at various times of the business cycle. Using the lens of multiscale entropy, a new understanding of the evolution of the yield curve and the economy can be gained.

MULTISCALE ENTROPY

(Costa, Goldberger, and Peng, 2002) introduced the use of multiscale entropy to study complex physiological processes such as the cardiovascular system. Declines in the information processing capabilities of biological systems often occur with aging and also in the presence of diseases. This reduction in the ability to process complex information is measured as a fall in entropy.

These biological applications of entropy ran into the limitations of single timescale entropic measures. Entropy can increase with disorder, and disorder rises in highly complex system and properly operating systems. However, there are also examples where diseased systems can also display high degrees of disorder such atrial fibrillation in heart failure. This made the use of measures of entropy at single timescales problematic. (Costa et al., 2002) found they were able to distinguish between diseased hearts and healthy cardiovascular systems by looking at entropy at the circulatory system at various scales (or multiscale entropy).

DEFINITION OF MSE

The concept of MSE as developed by (Costa et al., 2002) is composed of two procedures. First Costa described a method of coarse graining of the time series of interest. This is accomplished by averaging the data series with non-overlapping windows of increasing length τ. Next the sample entropy of each coarse-grained time series is calculated. These sample entropies are plotted as a function of the corresponding scale factor.

From (Costa et al., 2002:137) below:

"…We briefly describe the MSE method

Given a time series $\{x_1, \ldots, x_i, \ldots x_n\}$, we first construct consecutive coarse-grained time series by averaging a successively increasing number of data points in non-overlapping windows (Figure 1). Each element of the coarse-grained time series, $y_j^{(\tau)}$, is calculated accordingly to the equation:

$$y_j^{\tau} = 1/\tau \sum_{i=(j-1)\tau+1}^{j\tau} x_i \ldots\text{"}$$

MSE AND THE YIELD CURVE AS A MSE

(Costa et al., 2002) defined the MSE as composed of a measure of entropy at various timescales. The empirical original time series of the heartbeat for instance is used to derive estimates of the MSE at other smaller and larger timescales. As shown previously interest rates can be viewed as a measure of the growth rate of entropy in the economy. Additionally, the yield curve presents interest rates at various timescales (yields to maturity).

One important difference between the traditional MSE of (Costa et al., 2002) and that presented in this paper is the ready availability of yield curve data at various timescales. This fortunately means there is no need to derive the entropy at various timescales as is typical with the second step of the traditional MSE methods.

MSE LIFE CYCLE

In this paper the author extends the MSE into the concept of a continuous life cycle. Over time the MSE of the economy goes through an evolution of yield curve shapes. These shapes and their progression turn out to be a natural and intuitive evolution of the economy specifically and of a class of complex systems in general including living systems or organisms. There are other systems characterized by information processing that go through cycles or have a life cycle. These include brain structures and neural networks and the human heart and circulatory system. Just as with our economy, information processing is central to the operation and health of these systems. Multiscale entropy has been used extensively in the last few years to study the behavior and decline of functioning of a myriad of systems.

ENTROPY GROWTH: SHORT VS LONG TERM TIMESCALES

The amount of entropy in the economy can be measured by the area underneath the yield curve from (Parker, 2017, 2018, 2019, 2024). Over the business cycle that area tends to initially increase, reach a maximum, and decline until the start of the next cycle. The next cycle begins with a slightly greater ability to process information using the latest technology and organizational structures. This is evidenced by a spike or positive hump at the short end of the curve. The economy can then expand information processing at all timescales. This gradually raised the yield curve at greater and greater timescales until the maximum timescale is reached. Solving the entropic yield curve for the initial form at cycle start (t=1) is presented below (Note that the cycle start r (and R/C) is unchanging cycle to cycle if R and C grow by the same overall proportion between cycle starts):

$$r_1 = \left(-e^{-C_1\left(1-\frac{R}{C}\right)}\right)\ln(\sigma)$$

As timescales lengthen there are diminishing increases in entropy. Beyond a certain maximum timescale there will be a net reduction in entropy as the short end ultimately becomes overwhelmed with the increasing data streams and numbers of variables and relationships being utilized by the economy. Note information is collected and processed at all timescales, but the shortest timescales will hit their limits of communication and computation first.

The shortest timescales have the largest amount of data per second available which also represents the largest learning opportunity in terms of incoming information. There is still knowledge discovery happening at longer and longer timescales but at a slower rate. As the shorter timescales reach their limits a greater proportion of the total entropy growth will come from the remaining longer timescales. This shift in entropy production growth towards the long-term scales leads to an overall reduction in the rate of entropy production. This rate of reduction grows until entropy growth stops or falls below a critical level of diminishing marginal return in the entropy production rate. At this point the only way to increase the rate of entropy production is to begin a new cycle with a superior and faster information processing technology. At this point a new cycle begins with a more advanced technological base and increasing levels of entropy production at the new shorter timescales.

This means information processing timescales will move slightly to the left in the yield curve graph, expanding processing to newer and faster regimes. The increased information processing at the shortest scale will spread throughout all timescales and expand overall total entropy production. This process will continue until these new shortest scales hit their limits.

This increase in entropy at the shortest timescale is in contrast with the longer timescales which would add very little additional valuable information beyond the current maximum. This can be understood intuitively when we imagine the high value of being able to react to events happening around us on hourly, daily, or weekly timescales vs a twenty or thirty-year timescale events. It is hard to imagine much useful information to be gained at expanding beyond thirty years for individual human economic agents or even the economy as a whole. (Note: The leftward expansion will not actually be seen in a standard yield curve chart since it has a set minimum of 1 month, only the new hump at the short end will be visible).

THERMODYNAMIC UNCERTAINTY RELATIONS

(Barato and Seifert, 2015) derived the original Thermodynamic Uncertainty Relation (TUR) in 2015 as seen below. Many other forms and generalizations of the TUR have been developed. All derivations share the central unifying principle that there is a thermodynamic cost to the precision of thermodynamic machines and processes. Specifically, the precision or variance of a variable of interest J (produced by the thermodynamic machine) defines the minimum amount of entropy needed to maintain that precision.

$$\langle \sigma \rangle \geq \frac{2\langle J \rangle^2}{Var[J]}$$

In the thermodynamic economy this relationship implies that as entropy production decreases, the variance of interest rates will increase.

$$\langle \sigma \rangle \downarrow \rightarrow Var[J] \uparrow$$

In (Parker 2017, 2018, 2019, 2024) it was demonstrated that at the shortest timescales entropy production falls near the end of the business cycle. This fall in entropy production is predicted to be accompanied by an increase in the variance of the interest rates at those shortest timescales by the TURs. This implies that early in the business cycle interest rates will have relatively similar variances. Near the end of the cycle the shorter timescale rates should increase in variance relative to the longer timescales. In the following section this relationship is empirically studied over three business cycles.

RESULTS

In this section the variance of the United States yield curve at all available terms to maturity is studied over three business cycles. The starting dates of equity bear markets is used to define the end of the cycle. These three periods had bear market starts in 2007, 2020, and 2022 respectively. Figure 5 displays the variances of the complete yield curve over the entire sample period for perspective. Figures 6, 7, and 8 provide a more detailed view of the curves around the time of the bear market starts in 2007, 2020, and 2022 respectively.

All three sample periods have large variance spikes in the relatively shorter time to maturity curves. This behavior is consistent with the predictions from the TUR perspective. The notable exceptions are the 1-month and 2-month yield curves as seen in Figure 8 in 2022 which does not exhibit the same relatively large spikes as seen in the 1-year, 2-year, 3-

month, and 6-month yield curves (Note: the spikes seen are still relatively focused on the shorter end of the curve). This may be due to the historic anti-inflationary intervention in the bond market by the Federal Reserve beginning in 2022.

Figure 5: Variance of the Yield Curve over the Entire Sample Period (2007-2022).

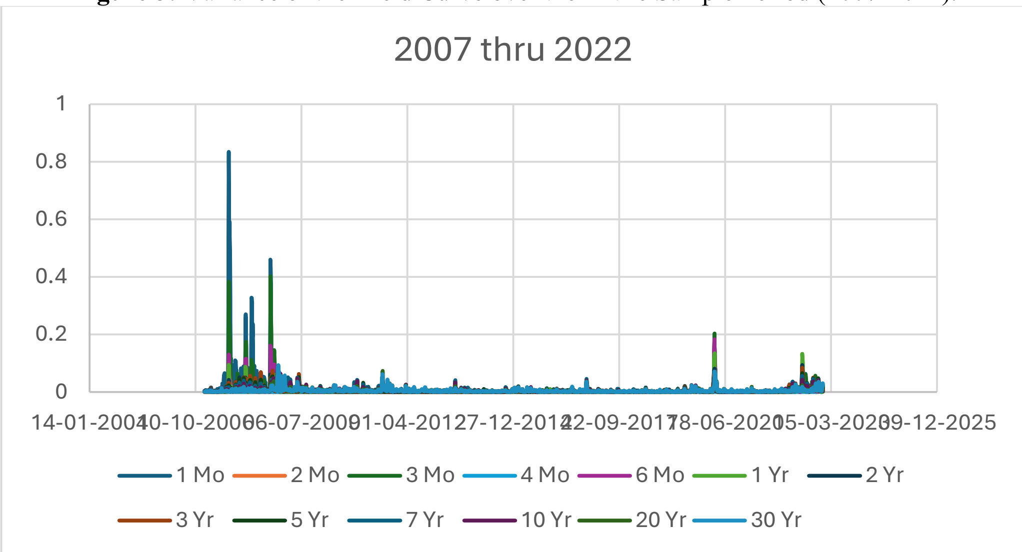


Figure 6: Yield Curve Variances: Spike of the 1 month, 3 month, and 6 month yield curve variances (2007).

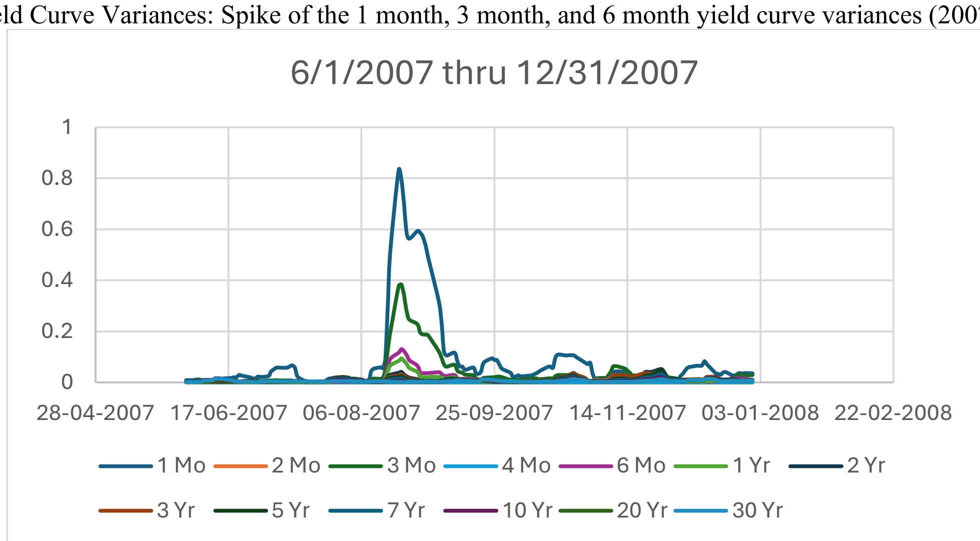


Figure 7: Yield Curve Variances: Spike of the 1 month, 2, month, 3 month, and 6 month yield curve variances (2020).

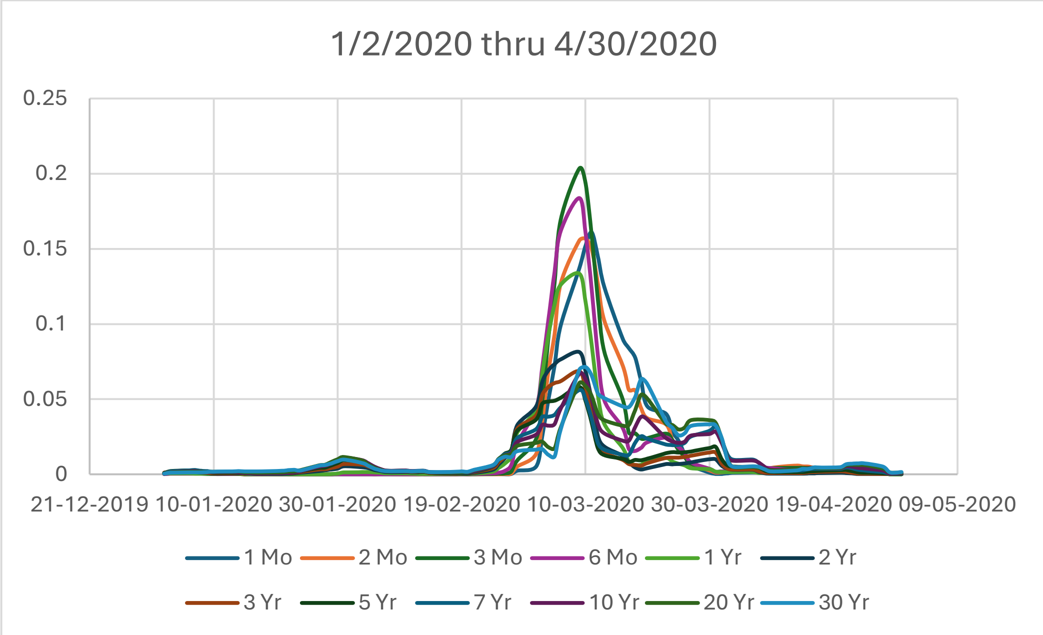
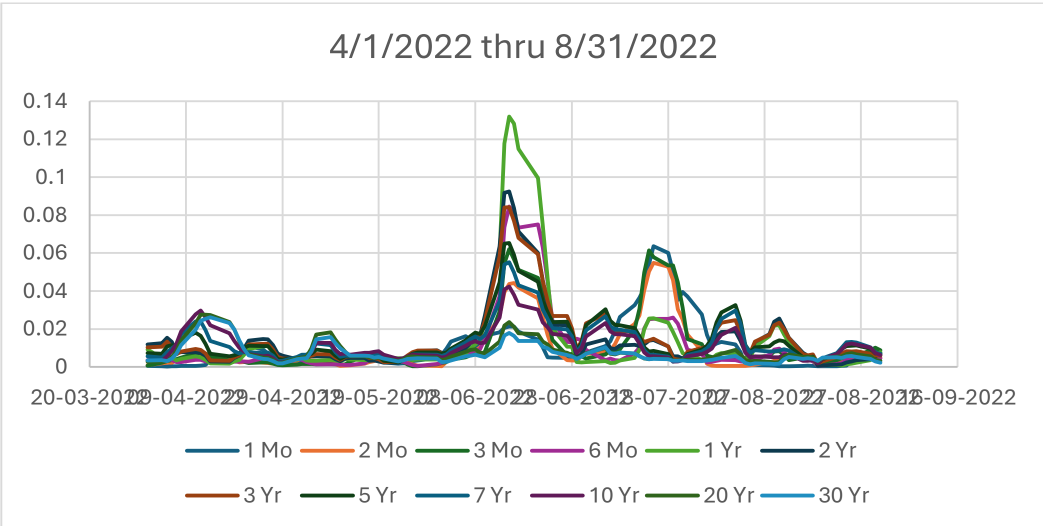


Figure 8: Yield Curve Variances: Spike of the 1 year, 2 year, month, 3 month, and 6 month yield curve variances (2022).



CONCLUSIONS

The recent derivations of the TURs have important implications for the stability and growth of the economy. The TURs demonstrate that the precision (or stability of the economic system or subsets of that system) defines the lower bound on the total entropy production of the economy. This relationship implied a divergence in the variances of short term versus long term rates at the end of business cycles as demonstrated previously. Specifically, it was found that the variance of short-term rates increases dramatically towards the end of the business cycle in comparison to long term rates consistent with the effect predicted by the TURs. Potential directions for future study include studying the impact of the TURs on other economic and financial variables and processes of interest.

Data Availability Statement: The historical yield curve rates used in this paper can be downloaded at www.treasury.gov and other background material is available at www.entropicfinance.com.

Conflicts of Interest: The authors declare no conflicts of interest.

REFERENCES

[1] Barato, A.C.; Seifert, U. (2015) Thermodynamic uncertainty relation for biomolecular processes *Physical review letters*, 114 (15), 158101.
[2] Baumgärtner, S. (2002) Thermodynamics of waste generation. 10.4337/9781035335572.00010.
[3] Chen, J. (2005) The Physical Foundation of Economics: An Analytical Thermodynamic Theory. 10.1142/5819.
[4] Costa, M.; Goldberger, A.L.; and Peng, C.K. (2002) Multiscale entropy analysis of complex physiologic time series. Phys. Rev. Lett., 89(6):068102, Aug 2002.
[5] Grisolia, G.; Fino, D.; Lucia, U. (2020) Thermodynamic optimization of the biofuel production based on mutualism. Energy Reports. 6. 1561-1571. 10.1016/j.egyr.2020.06.014.
[6] Kardes, G.; Wolpert, D.H. (2021) Thermodynamic Uncertainty Relations for Multipartite Processes. arXiv preprint arXiv:2101.01610, 2021. 3.
[7] Kümmel, R. (2011) The Second Law of Economics: Energy, Entropy, and the Origins of Wealth Springer, Berlin, ISBN 978-1-441-99364-9.
[8] Landauer, R. (1961) Irreversibility and Heat Generation in the Computational Process, IBM Journal of Research and Development, Vol. 5, No. 3, pp. 183-191. http://dx.doi.org/10.1147/rd.53.0183.
[9] Parker, E. (2017) The Entropic Linkage between Equity and Bond Market Dynamics. Entropy 19, 292.
[10] Parker, E. (2018) The Relationship between the US Economy's Information Processing and Absorption Ratios: Systematic vs Systemic Risk. *Entropy*, *20*, 662. https://doi.org/10.3390/e20090662.
[11] Parker, E. (2019) Entropy, the Information Processing Cycle, and the Forecasting of Bull and Bear Market Peaks and Troughs. *International Journal of Productivity Management and Assessment Technologies (IJPMAT), 7*(1), 77-90. http://doi.org/10.4018/IJPMAT.2019010105.
[12] Parker, E. (2024) *The Signal, Entropic Finance, and the Digital Economy: New Perspectives at the Intersection of Information Theory, Entropy, and Finance*. 2nd ed.; KDP Publishing United States; pp. 140-160. (Also, at https://papers.ssrn.com/abstract=3853703).
[13] Shannon, C.; Weaver, W. (1949) *The Mathematical Theory of Communication*; The University of Illinois Press: Chicago, IL, USA.

[i] Other authors such as (Kardes and Wolper, 2021) have generalized the TURs analysis to systems composed of multiple interacting parts. These results modify the end results slightly numerically without changing the broader conclusions.