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Mathematics of uncertain multi-body systems

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abstract

As a consequence, numerous branches of simulation technology are warming up to the idea of taking uncertainties into account in numerical simulation, which is both reasonable and frequently necessary for producing trustworthy results. However, uncertainties have only been sometimes taken into consideration in multibody system analysis. Uncertainties are often thought of as being of a random character, or aleatory uncertainties, which may be effectively managed by using probability theory. So-called epistemic uncertainties, such as those attributable to a lack of knowledge, to subjectivity in numerical implementation, and to simplification or idealization, actually account for a significant portion of the uncertainties built into dy amical systems in general, or multibody systems in particular. As a result, a suitable theory is needed to describe epistemic uncertainty in multibody systems, which is still a challenging problem. In light of this, an approach will be introduced that incorporates epistemic uncertainty into multibody system modelling and analysis. Based on fuzzy arithmetic, a subfield of fuzzy set theory, this strategy uses fuzzy numbers to represent the uncertain values of the model's parameters, which is a relatively straightforward and realistic representation of the fuzzy range of potential parameter values. By giving simulation results that account for the dynamics of the system as well as the impact of the uncertainties, this cutting-edge modelling approach allows for the derivation of more complete system models that surpass the conventional, crisp-parameterized models.

Introduction

Knowing the model parameters well is crucial in MBS modelling and simulation for producing results that are accurate representations of the real system's behaviour. The problem is that when models grow in complexity and richness, it gets harder to pin down their individual parameters. There may be considerable ambiguity in the indicated parameters, and it will be difficult to offer precise values for them. Moreover, assumptions of idealization and simplification are necessary to create a mathematical description with an appropriate number of degrees of freedom, despite the aforementioned growing focus on detail in modelling. As a result, despite appearances, even well-modelled MBS might display its inherent uncertainties as a result of modelling flaws, inaccurate data, or a lack of complete understanding. The lack of knowledge, fuzziness in parameter specification, and subjectivity in numerical implementation are all examples of the types of uncertainty that fall under the umbrella term "epistemic uncertainties" [1]. The analogue to this system is the term "aleatory uncertainty," which refers to the randomness or variation that occurs in nature. Probability theory, and in practice, often Monte Carlo techniques or polynomial chaos methods, are used to tackle aleatory uncertainty with success. When contrasted with this, extended modelling with epistemic uncertainty remains challenging from both a methodological and computational perspective. The notion of fuzzy set theory [2] has been gaining popularity in recent years as a method to describe epistemic difficulties. In this paper, we provide a novel interdisciplinary approach to system modelling and analysis that incorporates uncertainties, especially epistemic ones, from the outset of the modelling process. This strategy relies on a subfield of fuzzy set theory called fuzzy arithmetic, which has seen increased use with the development of the Transformation Method [3].

Uncertainty Classification

Even though there is a vast variety of uncertainty manifestations, the above classification into aleatory uncertainties and epistemic uncertainties is generally accepted and useful [1]. This classification is utilized throughout, despite the fact that alternative categories (such [4]) may be used in a nearly equivalent fashion. The following descriptions will elaborate on the various ideas and the scopes of their application.

Risks of the Aleatory Variety

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The time and space scattering of a system's physical attributes is an example of an aleatory uncertainty. They are haphazard and have to do with the unknown results of an event or an experiment. In this context, the use of random numbers with probability density functions obtained from measurements and experimental data might produce an effective representation of aleatoryrial uncertainties. Probability theory, Monte Carlo simulations, and polynomial chaos approaches are generally recognised in the literature (e.g. [5], [6]) as the most effective, adaptable, and often used methods for quantifying the propagation of the aleatory uncertainty across systems.

Doubts About Our Ability to Know

Lack of information, or epistemic uncertainty, may lead to problems like imprecise parameter specification, arbitrary numerical implementation, or oversimplification and idealization throughout the system modelling process, among other things. Probability theory may not be suitable for accurately representing epistemic uncertainty [7] due to the substantial and undeniably distinct nature of epistemic uncertainties compared to aleatory uncertainties. On top of that, real-world data for a randomness-based measurement of the uncertainties are often unavailable. For these reasons, this study instead adopts the approach of characterizing epistemic uncertainties using fuzzy numbers [2, 8], and employs fuzzy arithmetic [8, 9] to evaluate the model with fuzzy-valued parameters and therefore propagate the uncertainty throughout the system. If just worst-case boundaries and no other information about a potential distribution inside the interval is provided, then representing epistemic uncertainty by ordinary intervals seems to be the most practical and easy option. While the dependency problem (also known as the overestimation effect [9, 3]) makes it difficult to use classical interval arithmetic in the evaluation of models with interval-valued parameters, the sharp boundedness of the intervals acts quite counter to the prevailing human perception of quantifying imprecision. Instead, the ambiguous limits of fuzzy numbers are a better fit with this perspective. Furthermore, the question of how the results of the propagation will change (both qualitatively and quantitatively) with the amount of initial uncertainty, i.e., with the lengths of the intervals assumed, is automatically raised when uncertainty is propagated based on a single set of intervals for the uncertain parameters. This constraint is easily overcome by using fuzzy numbers, which may be seen as a series of nested intervals spanning from a worst-case scenario in the presence of maximal uncertainty to a crisp nominal value in the presence of total confidence (see Section 3).

Theory with some uncertainty as already said

Previously, we saw that fuzzy set theory works effectively for capturing epistemic ambiguities. Following this conceptual groundwork, we introduce how fuzzy arithmetic may be used for the numerical analysis of dynamical models.

Misleading Figures The numerical implementation of uncertain model parameters as fuzzy numbers [8] is a specific application of the theory of fuzzy sets that is somewhat distinct from the well-established usage of fuzzy set theory in fuzzy control. Fuzzy numbers may be thought of as convex fuzzy sets over the universal set R, with membership functions (x) [0, 1], where (x) = 1 holds true only for the single value x = x R, the so-called center value or nominal value. Here's an illustration: the triangular (linear) fuzzy number [9], written in the shortened form [9].

$$\tilde{p} = \mathrm{tfn}(\bar{x}, \alpha_l, \alpha_r) \; ,$$

is defined by the membership function

$$\mu_{\tilde{p}}(x) = \begin{cases} 0 & \text{for} \quad x \le \bar{x} - \alpha_l \\ 1 + (x - \bar{x})/\alpha_l & \text{for} \quad \bar{x} - \alpha_l < x < \bar{x} \\ 1 - (x - \bar{x})/\alpha_r & \text{for} \quad \bar{x} \le x < \bar{x} + \alpha_r \\ 0 & \text{for} \quad x \ge \bar{x} + \alpha_r \end{cases} .$$

However, if it's necessary to put a number on the uncertainty around a particular model parameter, a membership function with a different form may be used. Fuzzy arithmetic, the computation using fuzzy numbers, is a non-trivial issue, particularly when used to the assessment of complex mathematical models with fuzzy-valued operands. Please refer to Liu [10, 11] for a thorough exposition of the mathematical foundations of fuzzy variables within the context of credibility theory.

System Dynamics in a Multi-Body Environment

The multibody technique is often employed for the modelling of coupled rigid bodies that carry out substantial operating movements. Since MBS

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formulations may be used to such a broad variety of engineering issues, their study is a lively area of inquiry [26]. As a result of solving the Newton-Euler equations, we get the differential-algebraic equations of second order that describe the motion of the system (DAE). By replacing the reaction forces that relate the ODE with the algebraic equations with generalized coordinates y, we may reduce these nonlinear equations to a minimum form representation, i.e., a set of ordinary differential equations. Initial conditions on location and velocity, y0 and y 0, and the equations of motion themselves may both be affected by the parameter dependence in the most generic situation. The parametric system may be expressed in its simplest form as follows.

$$M(y;\overline{p})\,\ddot{y} + k(y,\dot{y};\overline{p}) = q(y,\dot{y};\overline{p}),$$
$$y\,(t=0;\overline{p}) = y_0(\overline{p}), \quad \dot{y}\,(t=0;\overline{p}) = \dot{y}_0(\overline{p}).$$

Minimum mass matrix denoted by M, generalized centrifugal, Coriolis, and gyroscopic forces denoted by k, and generalized applied forces denoted by q. The ep parameter vector stores all the fuzzy parameters together. The formation of the motion equations may be performed by several approaches, including the use of absolute or relative kinematics, as well as numerical or symbolic formulations. A number of evaluations of Equation 3 for the system in various perturbed states, i.e., different locations in the parameter space, are needed as part of the solution technique for obtaining the fuzzy valued output. A symbolic formulation is useful for assessing parametric uncertainties since the equations of motion need to be established once. [27] Neweul-M2 is a tool for MBS simulations that uses symbols.

Model Order and Elasticity in the MBS Elasticity Reduction

If the bodies undergo non-trivial elastic deformations, then the large working motion must be calculated to account for the effect of the deformation. Nonlinear finite element methods may do this by include parameters for slope and rotation in addition to the ansatz functions representing the deformation field. This method is used, for example, in the big rotation vector formulation [28, 29] and the absolute nodal coordinate formulation [27, 28]. The floating frame of reference method offers an alternative and, in general, more economical method for emulation if the deflections fall within the range of linear elasticity. In this case, we employ the concept of reference frames to represent the massive mobility of the workers. The deformation is subsequently taken into account in the local coordinate system of the reference frame, and the elastic bodies are coupled to the frame. A point's location on an elastic body may be broken down into the reference position, the location of the point in the reference frame, and the deflection. It is still possible to write the system equations in the general form of Equation 3, but with the inclusion of the elastic coordinates at y.

Several software packages exist to simulate elastic bodies using the finite element method, which may be used for a broad variety of tasks. While the resulting system dimension is often too large to be computed within the elastic MBS framework, the dynamic behaviour of the system in the frequency range of interest may be represented by a relatively compact system that is attainable via model reduction. Modern reduceton approaches based on balanced truncation or moment matching have acquired substantial popularity in recent years [30], in addition to the older modal reduction techniques such as component modes synthesis. If the quantities linked to rigid bodies or joints are all that need to be considered for the propagation of uncertainty, then the same methods may be used as previously. Therefore, in this scenario, the addition of leas tic bodies does not increase the difficulty of quantifying uncertainty.

Propagation of uncertainty may be managed in the same way as stated above if they are confined to values associated with rigid bodies or joints. Accordingly, the addition of leas tic bodies does not add complexity to the measurement of uncertainty in this instance. However, the situation becomes more complicated if unknown factors are linked to individual building components. It is feasible to solve this problem by computing an elastic MBS for each point in the parameter space, which entails establishing the system matrices of the FE model, performing a model reduction, and then deriving the equations of motion of the newly constructed elastic MBS. Obviously, this method is too time-consuming to be useful for complicated systems. Model reduction that takes into consideration the parametric dependence and retains it in the final reduced model is more effective but more methodologically challenging. This is a newly popularized area of study called as parametric model order reduction (pMOS). Most pMOR methods, at their core, interpolate between numerous reduced-order models to arrive at a representation of the full-order model. As a result, the parameter dependence is preserved in the simplified model, and it is once again possible to

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operate inside a symbolic framework without resorting to a recalculation of the elastic MBS equations.

MBS Uncertainties Apart

In addition to the well-studied aleatory uncertainties, such as the fluctuation of material characteristics and geometrical parameters owing to abnormalities in manufactureIng or assembly, the epistemic uncertainties play a significant role in MBS modelling. These doubts in knowledge may be broken down further into the following classes according to their source and nature: Having a wide range of unknown or poorly defined operating circumstances (including beginning conditions, applied loads, friction model parameters, etc.)

Implementation subjectivity includes, but is not limited to, the use of varying integration schemes, time steps, model order reduction methods, the number of reduced basis vectors, and other numerical evaluation techniques. • Inadequacies in the models themselves, such as the idealization or simplification of models to assure or expedite the numerical assessment, or the use of idealized or simplified constitutive laws for material models (as is done for composite materials, friction behaviour, etc.). The modelling process of MBS is susceptible to epistemic uncertainties of all three types. However, in the context of comprehensive modelling—that is, modelling both the system and possible uncertainties—these uncertainties can be successfully represented and quantified by fuzzy numbers, and the propagation of the uncertainties through the model can be evaluated with the help of the Transformation Method of fuzzy arithmetic.

Example

The following case study will show how to model and analyse an MBS thoroughly by taking into account uncertainties expressed as fuzzy-valued parameters. A planar two-link manipulator with motors at both joints makes up the system. Figure 2 shows a drawing of the proposed model. As shown, the angles 1 and 2 are each one degree of freedom, and the torques T1 and T2 are applied by motors at the joints. The linkages also have viscous and friction damping, implemented using the Strobeck friction model. Joint i's damping moment, I am calculated as

$$\phi_i(\dot{\theta}_i) = d\,\dot{\theta}_i + \phi_C(\dot{\theta}_i;\mu_C) + \left(\phi_s(\dot{\theta}_i;\mu_C) - \phi_C(\dot{\theta}_i;\mu_C)\right) e^{-\left(\dot{\theta}_i/v_S\right)^2},$$

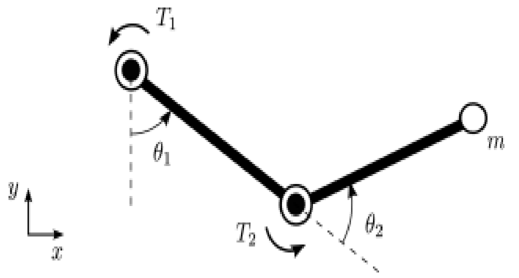

Fig.1a representation of a two-link manipulator model

involving the viscous damping coefficient d, the Coulomb friction damping moment C, which is proportional to the friction coefficient C, and the static friction damping moment s, which is proportional to the stiction coefficient s. The ratio of the static friction value to the Coulomb friction value decreases exponentially with increasing rotational velocity, and this ratio is affected by a scaling factor termed the Strobeck velocity, or vs. The second arm has a weighted end effector at its tip, designed to be propelled in a predetermined direction. To achieve this goal, a feedforward control was determined using the nominal model, with the aim of having the actuated joints move the end effector to the appropriate location. xref = h x(t)ref, y(t)refit is the end effector's time-dependent position vector, which represents the reference trajectory.

The driven trajectory will only depart from the planned trajectory by a tiny amount if the model parameters are known to within a high degree of accuracy. Now we must take into account the uncertainty around the model's inputs. In particular, m, the mass of the end effector, is not known with any degree of accuracy. This may be because of the fact that a variety of tool tips may be attached to the end effector (lack of knowledge). Further, Equation 4 describes a refined, but still simple, model of friction damping. This might be thought of as a degree of doubt stemming from imperfect modelling, which may stem, in part, from a lack of information about the relevant parameters. In Section 3.1, Equation 2 defines triangular fuzzy numbers as a means of characterizing the unknown parameters, and this is the form used to represent the parameters in this section (pie = tfn (xi, l, i, r, i)). Table 1 displays the nominal values xi and the worst-case deviations lei and r, i.

table. 1 we define several fuzzy parameters.

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	$\tilde{p}_1 := \tilde{d}$	$\tilde{p}_2 := \tilde{\mu}_s$	$\tilde{p}_3 := \tilde{\mu}_C$	$\tilde{p}_4 := \tilde{v}_S$	$\tilde{p}_5 := \tilde{m}$
$\bar{x}_i$	0.05 $\frac{kg}{s}$	0.4	0.3	0.1 $\frac{rad}{s}$	1.0 kg
$\alpha_{l,i}/\bar{x}_i$	2.5%	2.5%	2.5%	2.5%	1.0%
$\alpha_{r,i}/\bar{x}_i$	2.5%	2.5%	2.5%	5.0%	1.0%

At some point, when these unknowns are included into the analysis, the manipulator will no longer be able to precisely retrace the route for all possible configurations, and instead will deviate significantly from the original path. In order to put a number on this impact, we run a simulation of the system using the Transformation Method. Figure 3 depicts the resultant fuzzy-valued trajectory, in which the end-motion effector's is shown in the x and y directions and the membership value of solutions is represented by colouring. To find the solution of the nominal system that most closely matches the supplied trajectory, look for the black curve within the collection of solutions.

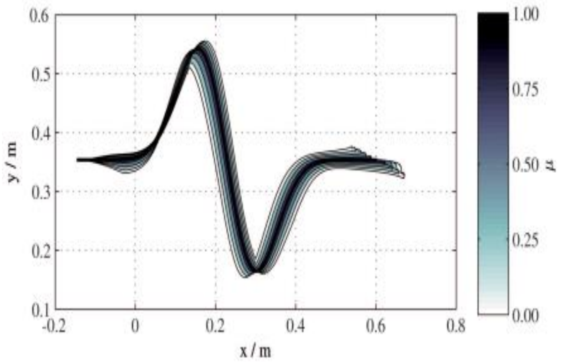

Figure 2: A fuzzy answer regarding how the feedforward system's end-effector should move. Nominal system solutions are shown in black, and their membership values are shown on a contour plot. Membership values are shown in the color scale to the right.

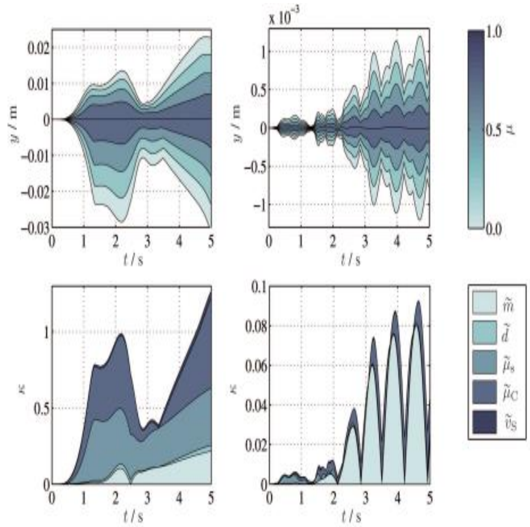

Figure 1: Open-loop (left) and closed-loop (right) system error (top) and influence (bottom) estimates for end-effector motion in the y-axis (right). (Take into account the varying degrees of magnitude.)

Applying a control, such a PD-control, may help mitigate the effects of fluctuations in input data due to the uncertainty in the model's parameters.

$$T_i = -k_{pi}\left(\theta_{ref,i} - \theta_i\right) - k_{di}\left(\dot{\theta}_{ref,i} - \dot{\theta}_i\right), \quad i \in \{1,2\},$$

where ref, i = ref, i(x(t)ref, y(t)ref) and ref, i = ref, i(x(t)ref, y(t)ref, x (t)ref, y (t)ref) are the reference angles and angular velocities, respectively, that follow from the construction of the reference trajectory. Torques are employed directly as control outputs without any consideration of motor dynamics. As shown in Figure 4, both the open-loop (left) and closed-loop system's y-motion end effector errors (top) and influence measurements I (bottom) are displayed (right). Both the inaccuracy and the absolute effect are drastically decreased by the controller, as seen in the figures. Since the friction moments have an impact on the driving torques directly, they are efficiently compensated by the PD-control, making the combined effects of damping and friction insignificant in the closed-loop situation. The oscillatory behaviour makes it clear that the bulk of the residual output uncertainty is caused by inertial effects, while mass variation also plays a role. For lower membership levels, the error limits of the closed-loop system slowly expand, indicating unstable behaviour; nevertheless, for % 3 4 at least, the error remains limited.

This means the controller has a relatively limited resilience zone relative to model uncertainty.

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(Remember, we haven't factored in disturbances yet.) If you want, you can easily determine the threshold value of the membership level for which stability is guaranteed, and thus obtain explicit margins of robustness, i.e., the maximum tolerable deviation of each model parameter from its nominal value. Parameters that are critical to the system's resilience may be recognized and should be identified as precisely as possible with the help of the supplemental information supplied by the influence measurements. The time intervals or frequency ranges in which an identification of a model parameter may be achieved more reliably are clearly assignable, since the impact measurements can be computed with regard to time, frequency, or any other independent variable. This paves the way for ignoring measurement data in cases when there is insufficient information for an identification. When using the two-arm manipulator's output as an example, it is not possible to accurately determine the value of the viscous damping factor d; nevertheless, the stiction coefficient s exhibits significant effect across the whole time period and may be accurately recognized.

Conclusion

Modelling, solving, and analysing issues in multibody dynamics while taking into account uncertainties has been shown to benefit greatly from the use of fuzzy arithmetic based on the Transformation Method. By doing so, the robustness against uncertainties of various designs, including the robustness assessments of applicable controllers, may be compared and contrasted. It is also possible to quantify the impact of each unknown parameter separately. The Transformation Method is an adaptable, generalized process that may be used for a broad variety of issues with just modest modifications. Although there are unique challenges involved in resolving fuzzy-parameterized multi-body systems, some of which have been touched on above, there are still others that need to be addressed. The increasing computing effort is one of the limiting aspects of calculations considering uncertainty in complex systems.

The amount of work required is directly related to the number of unknown parameters, since the computational complexity of most existing computational techniques rises exponentially with the dimension of the parameter space. For example, if you want to sample the edges of a hypercube, you'll need twice as many points in each dimension. That's why it's called "the curse of dimensionality," or the negative effects of having more space than you need. Therefore, it is of paramount importance to find ways to speed up the computations itself, such as by decreasing the number of required model assessments. Further research on pMOR techniques is also necessary for the effective computation of systems with uncertainty in elastic bodies.

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