

A Look at the Development of Physics and Its Social Role

¹Dr. SANJEEV KUMAR, Prof, M.Sc., Ph.D,

sknijalinge@gmail.com,

²MUDIGONDA NAGA RAJU, Asst. Prof, M.Sc.,

nraj.mudhigonda@gmail.com,

³M SRAVAN KUMAR, Asst. Prof, M.Sc.,

sravanvadavphysics@gmail.com,

⁴S SWAPNA RANI, Asst. Prof, M.Sc.,

swapnasangireddy@gmail.com,

Department of H&S,

Pallavi Engineering College,

Kuntloor(V), Hayathnagar(M), Hyderabad, R.R. Dist. -501505.

Abstract

This paper analyzes the recent growth in our understanding of physics in general and quantum world in particular focusing on its relevance to our society. Physics touches every aspect of our lives. It involves the study of matter, energy and their interactions. As such, it is one area of science that cuts across all other subjects. Other sciences are reliant on the concepts and techniques developed through physics. Other disciplines — such as chemistry, agriculture, environmental and biological sciences — use the laws of physics to better understand the nature of their own studies. Physics focuses on the general nature of the natural world, generally through a mathematical analysis. Physics is one of the most difficult subjects taught in schools. A number of students are even more daunted with its use of mathematics.

In a study done in UK from 1985 to 2006, it was found that there was 41 percent decrease in the number of entries to A-level examinations in sciences. This decreasing trend is similar in other countries. Despite this trend, physics remains an integral part of the educational system. It is through physics that new methodologies were developed that helped improve the quality of life, including things such as automobiles and modern construction. Society's reliance on technology represents the importance of physics in daily life. Many aspects of modern society would not have been possible without the important scientific discoveries made in the past. These discoveries became the foundation on which current technologies were developed. Discoveries such as magnetism, electricity, conductors and others made modern conveniences, such as television, computers, phones and other business and home technologies possible. Modern means of transportation, such as aircraft and telecommunications, have drawn people across the world closer together — all relying on concepts in physics. In 1999 during the World Conference on Science (WCS), the UNESCO-Physics Action Council considered physics an important factor in developing solutions to both energy and environmental problems.

Physics seeks to find alternative solutions to the energy crisis experienced by both first world and developing nations. As physics help the fields of engineering, bio-chemistry and computer science, professionals and scientists develop new ways of harnessing preexisting energy sources and utilizing new ones. In the United Nations Millennium Summit held in 2000, it was recognized that physics and the sciences will play a crucial role in attaining sustainable development. Physics helps in maintaining and developing stable economic growth since it offers new technological advances in the fields of engineering, computer science and even biomedical studies.

Key words: physics, United Nations Millennium Summit, biomedical, computer science, engineering

Introduction

These fields play a crucial role on the economic aspect of countries and finding new and better ways to produce and develop products in these fields can help boost a country's economy. Similarly, the International Union of Pure and Applied Physics (IUPAP) asserted that physics will generate the necessary knowledge that will lead in the development of engines to drive the world's economies. In Rwanda, the education ministry was mandated to develop the country's scientific and technical know-how. Medical physics and information technology benefited the country by developing a national nutrition program and an epidemic surveillance system. Physics and engineering helped rural areas gain safe drinking water through gravimetric techniques, irrigation techniques and rainwater harvesting. In recent years, although physics has not experienced the sort of revolutions that took place during the first quarter of the twentieth century, the seeds planted at that time are still bearing fruit and continue to engender new developments.

We should keep in mind that nature is unified and has no borders; practical necessity has led humans to establish them, creating disciplines that we call physics, chemistry, biology, mathematics, geology, and so on. However, as our knowledge of nature grows, it becomes increasingly necessary to go beyond these borders and unite disciplines. It is essential to form groups of specialists—not necessarily large ones—from different scientific and technical fields but possessed of sufficient general knowledge to be able to understand each other and collaborate to solve new problems whose very nature requires such collaboration. Physics must be a part of such groups. We find examples of interdisciplinarity in almost all settings, one of which would be the processes underlying atmospheric phenomena. These involve all sorts of sciences: energy exchanges and temperature gradients, radiation received from the Sun, chemical reactions, the composition of the atmosphere, the motion of atmospheric and ocean currents, the biology of animals and plants that explains the behavior and reactions of animal and plant species, industrial processes, social modes and mechanisms of transportation, and so on and so on. Architecture and urban studies offer similar evidence.

For climate change, energy limitations, atmospheric pollution, and agglomeration in gigantic cities, it is imperative to deepen cooperation among architecture, urban studies, science, and technology without overlooking the need to draw on other disciplines as well, including psychology and sociology, which are linked to the study of human character. We must construct buildings that minimize energy loss, and attempt to approach energy sufficiency, that is, sustainability, which has become a catchword in recent years. Fortunately, along with materials with new thermal and acoustic properties, we have elements developed by science and technology, such as solar panels, as well as the possibility of recycling organic waste.

Objective:

This paper intends to study the advances theories in physics in 21st century and quantum mechanics great discoveries that have helped societal systems.

Growth and Public Interest in Physics

Physics is considered the queen of twentieth-century science, and rightly so, as that century was marked by two revolutions that

drastically modified its foundations and ushered in profound socioeconomic changes: the special and general theories of relativity (Albert Einstein, 1905, 1915) and quantum physics, which, unlike relativity, cannot be attributed to a single figure as it emerged from the combined efforts of a large group of scientists. Now, we know that revolutions, whether in science, politics, or customs, have long-range effects that may not be as radical as those that led to the initial break, but can nonetheless lead to later developments, discoveries, or ways of understanding reality that were previously inconceivable. That is what happened with physics once the new basic theories were completed. In the case of quantum physics, we are referring to quantum mechanics (Werner Heisenberg, 1925; Paul Dirac, 1925; Erwin Schrödinger, 1926). In Einstein's world, relativistic cosmology rapidly emerged and welcomed as one possible model of the Universe the experimental discovery of the Universe's expansion (Edwin Hubble, 1929). Still, the most prolific "consequences-applications" emerged in the context of quantum physics. In fact, there were so many that it would be no exaggeration to say that they changed the world. There are too many to enumerate here, but it will suffice to mention just a few: the construction of quantum electrodynamics (c. 1949), the invention of the transistor (1947), which could well be called "the atom of globalization and digital society", and the development of particle physics (later called "high-energy physics"), astrophysics, nuclear physics, and solid-state or "condensed matter" physics. One of the most celebrated events in physics during the last decade was the confirmation of a theoretical prediction made almost half a century ago: the existence of the Higgs boson.

Let us consider the context that led to this prediction. High-energy physics underwent an extraordinary advance with the introduction of particles whose names were proposed by one of the scientists responsible for their introduction: Murray Gell-Mann. The existence of these quarks was theorized in 1964 by Gell-Mann and George Zweig. Until they appeared, protons and neutrons had been considered truly basic and unbreakable atomic structures whose electric charge was an indivisible unit. Quarks did not obey this rule, as they were assigned fractional charges.

Subatomic world

According to Gell-Mann and Zweig, hadrons—the particles subject to strong interaction—are made up of two or three types of quarks and antiquarks called u (up), d (down) and s (strange), whose electric charges are, respectively, $2/3$, $1/3$, and $1/3$ of an electron's (in fact, there can be two types of hadrons: baryons—protons, neutrons, and hyperions—and mesons, which are particles whose masses have values between those of an electron and a proton).

Thus, a proton is made up of two u quarks and one d, while a neutron consists of two d quarks and one u. They are, therefore, composite structures. Since then, other physicists have proposed the existence of three more quarks: charm (c; 1974), bottom (b; 1977) and top (t; 1995). To characterize this variety, quarks are said to have six flavors. Moreover, each of these six can be of three types, or colors: red, yellow (or green), and blue. Moreover, for each quark there is an antiquark. (Of course, names like these—color, flavor, up, down, and so on—do not represent the reality we normally associate with such concepts, although in some cases they have a certain logic, as is the case with color). The combination of these earlier theories constituted a theoretical framework for understanding what nature is made of, and it turned out to have extraordinary predictive capacities. Accordingly, two ideas were accepted: first, that elementary particles belong to one of two groups—bosons or fermions, depending on whether their spin is whole or fractional (photons are bosons, while electrons are fermions)—that obey two different statistics (ways of

“counting” groupings of the same sort of particle). These are the Bose-Einstein statistic and the Fermi-Dirac statistic. Second, that all of the Universe’s matter is made up of aggregates of three types of elementary particles: electrons and their relatives (particles called muons and taus), neutrinos (electronic, muonic, and tauonic neutrinos), and quarks, as well as the quanta associated with the fields of the four forces that we recognize in nature (remember that in quantum physics the wave-particle duality signifies that a particle can behave like a field and vice versa): the photon for electromagnetic interaction, Z and W particles (gauge bosons) for weak interaction, gluons for strong interaction, and, while gravity has not yet been included in this framework, supposed gravitons for gravitational interaction.

Modern world quantum chromodynamics

The subgroup formed by quantum chromodynamics and electroweak theory (that is, the theoretical system that includes relativist theories and quantum theories of strong, electromagnetic, and weak interactions) is especially powerful, given the balance between predictions and experimental proof. This came to be known as the Standard Model, but it had a problem: explaining the origin of the mass of the elementary particles appearing therein called for the existence of a new particle, a boson whose associated field would permeate all space, “braking,” so to speak, particles with mass so that, through their interaction with the Higgs field, they showed their mass (it particularly explains the great mass possessed by W and Z gauge bosons, as well as the idea that photons have no mass because they do not interact with the Higgs boson). The existence of such a boson was predicted, theoretically, in three articles published in 1964—all three in the same volume of Physical Review Letters.

The first was signed by Peter Higgs (1964a, b), the second by François Englert and Robert Brout (1964), and the third by Gerald Guralnik, Carl Hagen, and Thomas Kibble (1964a). The particle they predicted was called “Higgs boson.” One of the most celebrated events in physics during the last decade was the confirmation of a theoretical prediction made almost half a century ago: the existence of the Higgs boson. Detecting this supposed particle called for a particle accelerator capable of reaching sufficiently high temperatures to produce it, and it was not until many years later that such a machine came into existence. Finally, in 1994, CERN approved the construction of the Large Hadron Collider (LHC), which was to be the world’s largest particle accelerator, with a twenty-seven-kilometer ring surrounded by 9,600 magnets of different types. Of these, 1,200 were two-pole superconductors that function at minus 217.3°C, which is even colder than outer space, and is attained with the help of liquid helium. Inside that ring, guided by the magnetic field generated by “an escort” of electromagnets, two beams of protons would be accelerated until they were moving in opposite directions very close to the speed of light. Each of these beams would circulate in its own tube, inside of which an extreme vacuum would be maintained, until it reached the required level of energy, at which point the two beams would be made to collide. The theory was that one of these collisions would produce Higgs bosons. The most serious problem, however, was that this boson almost immediately breaks down into other particles, so detecting it called for especially sensitive instruments. The detectors designed and constructed for the LHC are called ATLAS, CMS, ALICE, and LHCb, and are towering monuments to the most advanced technology. Following construction, the LHC was first tested by circulating a proton beam on September 10, 2008. The first proton collisions were produced on March 30, 2010, producing a total energy of 7·10¹² eV (that is, 7 tera-electron volts; TeV), an energy never before reached by any particle accelerator. Finally, on July 4, 2012, CERN publicly announced that it had detected a particle with an approximate mass of 125·10⁹ eV (or 125-giga-electron volts; GeV) whose properties strongly suggested that it was a Higgs boson (the Standard Model does not predict its mass).

This was front-page news on almost all newspapers and news transmissions around the world. Almost half a century after its theoretical prediction, the Higgs boson's existence had been confirmed. It is therefore no surprise that the 2013 Nobel Prize for Physics was awarded to Peter Higgs and François Englert "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider," as the Nobel Foundation's official announcement put it. Clearly, this confirmation was cause for satisfaction, but there were some who would have preferred a negative outcome—that the Higgs boson had not been found where the theory expected it to be (that is, with the predicted mass). Their argument, and it was a good one, was expressed by US theoretical physicist and proponent Jeremy Bernstein (2012 a, b: 33) shortly before the discovery was announced.

God particle triumph Higgs boson

Nonetheless, the fact, and triumph, is that the Higgs boson does exist, and has been identified. But science is always in motion, and, in February 2013, the LHC stopped operations in order to make adjustments that would allow it to reach 13 TeV. On April 12, 2014, it began its new stage with the corresponding proton-collision tests. This involved seeking unexpected data that reveal the existence of new laws of physics. For the time being, however, we can say that the Standard Model works very well, and that it is one of the greatest achievements in the history of physics, an accomplishment born of collective effort to a far greater degree than quantum mechanics and electrodynamics, let alone special and general relativity. Despite its success, however, the Standard Model is not, and cannot be "the final theory." First of all, it leaves out gravitational interaction, and second, it includes too many parameters that have to be determined experimentally. These are the fundamental, yet always uncomfortable whys. "Why do the fundamental particles we detect even exist? Why are there four fundamental interactions, rather than three, five or just one? And why do these interactions exhibit the properties (such as intensity and range of action) they do?" In the August 2011 issue of the American Physical Society's review, *Physics Today*, Steven Weinberg (2011: 33) reflected upon some of these points, and others: Of course, long before the discovery of neutrino masses, we knew of something else beyond the standard model that suggests new physics at masses a little above 10¹⁶ GeV: the existence of gravitation. And there is also the fact that one strong and two electroweak coupling parameters of the standard model, which depends only logarithmically on energy, seem to converge to get a common value at an energy of the order of 10¹⁵ GeV to 10¹⁶ GeV.

There are lots of good ideas on how to go beyond the standard model, including supersymmetry and what used to be called string theory, but no experimental data yet to confirm any of them. Even if governments are generous to particle physics to a degree beyond our wildest dreams, we may never be able to build accelerators that can reach energies such as 10¹⁵ to 10¹⁶ GeV. Some day we may be able to detect high-frequency gravitational waves emitted during the era of inflation in the very early universe, that can tell us about physical processes at very high energy. In the meanwhile, we can hope that the LHC and its successors will provide the clues we so desperately need in order to go beyond the successes of the past 100 years. Ultimately, quarks have color but hadrons do not: they are white. The idea is that only the "white" particles are observable directly in nature. Quarks are not, as they are "confined," that is, associated to form hadrons. We will never be able to observe a free quark. Now, in order for quarks to remain confined, there must be forces among them that differ considerably from electromagnetic or other forces. As Gell-Mann

put it (1995: 200): “Just as the electromagnetic force among electrons is measured by the virtual exchange of photons, quarks are linked to each other by a force that arises from the exchange of other types: gluons (from the word, glue) bear that name because they stick quarks together to form observable white objects such as protons and neutrons.”

Physics is considered the queen of twentieth-century science, and rightly so, as that century was marked by two revolutions that drastically modified its foundations and ushered in profound socioeconomic changes: the special and general theories of relativity and quantum physics. About a decade after the introduction of quarks, a new theory—quantum chromodynamics—emerged to explain why quarks are so strongly confined that they can never escape from the hadronic structures they form. Coined from *chromos*, the Greek word for color, the term chromodynamics alludes to the color of quarks, while the adjective quantum indicates that it meets quantum requirements. Quantum chromodynamics is a theory of elementary particles with color, which is associated with quarks. And, as these are involved with hadrons, which are the particles subject to strong interaction, we can affirm that quantum chromodynamics describes that interaction.

Search for quantum theories

So quantum electrodynamics and quantum chromodynamics function, respectively, as quantum theories of electromagnetic and strong interactions. There was also a theory of weak interactions (those responsible for radioactive processes such as beta radiation, the emission of electrons in nuclear processes), but it had some problems. A more satisfactory quantum theory of weak interaction arrived in 1967 and 1968, when US scientist Steven Weinberg and British-based Pakistani scientist Abdus Salam independently proposed a theory that unifies electromagnetic and weak interactions. Their model included ideas proposed by Sheldon Glashow in 1960. The Nobel Prize for Physics that Weinberg, Salam, and Glashow shared in 1979 reflects this work, especially after one of the predictions of their theory—the existence of “weak neutral currents”—was corroborated experimentally in 1973 at CERN, the major European high-energy laboratory.

Electroweak theory unified the description of electromagnetic and weak interactions, but would it be possible to move further along this path of unification and discover a formulation that would also include the strong interaction described by quantum chromodynamics? The answer arrived in 1974, and it was yes. That year, Howard Georgi and Sheldon Glashow introduced the initial ideas that came to be known as Grand Unification Theories (GUT). The detection of gravitational waves also reveals one of the characteristics of what is known as Big Science: the article in which their discovery was proclaimed (Abbott et al., 2014) coincided with the LIGO announcement on February 11 and was signed by 1,036 authors from 133 institutions (of its sixteen pages, six are occupied by the list of those authors and institutions). The importance of LIGO’s discovery was recognized in 2014 by the awarding of the Nobel Prize for Physics in two parts. One half of the prize went to Rainer Weiss, who was responsible for the invention and development of the laser interferometry technique employed in the discovery. The other half was shared by Kip Thorne, a theoretical physicist specialized in general relativity who worked alongside Weiss in 1975 to design the project’s future guidelines and remains associated with him today; and Barry Barish, who joined the project in 1994 and reorganized it as director. (In 2014, this prize had been awarded to David Thouless, Duncan Haldane, and Michael Kosterlitz, who used techniques drawn from a branch of mathematics known as topology to demonstrate the existence of previously unknown states, or “phases,” of matter, for example, superconductors and superfluids, which can exist in thin sheets—something previously considered

impossible. They also explained “phase transitions,” the mechanism that makes superconductivity disappear at high temperatures.)

Conclusion

Advances in the field of physics and related field of nanotechnoscience have made it possible to develop nanomaterials and nanodevices that are already being used in a variety of settings. It is, for example, possible to detect and locate cancerous tumors in the body using a solution of 35 nm nanoparticles of gold, as carcinogenic cells possess a protein that reacts to the antibodies that adhere to these nanoparticles, making it possible to locate malignant cells. In fact, medicine is a particularly appropriate field for nanotechnoscience, and this has given rise to nanomedicine.

The human fondness for compartmentalization has led this field to frequently be divided into three large areas: nanodiagnostics (the development of image and analysis techniques to detect illnesses in their initial stages), nanotherapy (the search for molecular-level therapies that act directly on affected cells or pathogenic areas), and regenerative medicine (the controlled growth of artificial tissue and organs). One particularly important example in which physics is clearly involved is the “Brain Activity Map Project”. And physics, like all of the other sciences, will play an important and fascinating role in shaping that future.

References

1. Fourier, Jean Baptiste Joseph (1822), *Théorie analytique de la chaleur* (in French), Chez Firmin Didot, père et fils
2. Grassmann, Hermann (1844), *Die Lineale Ausdehnungslehre - Ein neuer Zweig der Mathematik* (in German), O. Wigand, reprint: Hermann Grassmann. Translated by Lloyd C. Kannenberg. (2000), Kannenberg, L.C. (ed.), *Extension Theory*, Providence, R.I.: American Mathematical Society, ISBN 978-0-8218-2031-5
3. Hamilton, William Rowan (1853), *Lectures on Quaternions*, Royal Irish Academy
4. Möbius, August Ferdinand (1827), *Der Barycentrische Calcul : ein neues Hülfsmittel zur analytischen Behandlung der Geometrie* (Barycentric calculus: a new utility for an analytic treatment of geometry) (in German), archived from the original on 2006-11-23
5. Moore, Gregory H. (1995), "The axiomatization of linear algebra: 1875–1940", *Historia Mathematica*, 22 (3): 262–303, doi:10.1006/hmat.1995.1025
6. Peano, Giuseppe (1888), *Calcolo Geometrico secondo l'Ausdehnungslehre di H. Grassmann preceduto dalle Operazioni della Logica Deduttiva* (in Italian), Turin
7. Peano, G. (1901) *Formulario mathematico: vct axioms* via Internet Archive
8. Further references
9. Ashcroft, Neil; Mermin, N. David (1976), *Solid State Physics*, Toronto: Thomson Learning, ISBN 978-0-03-083993-1
10. Atiyah, Michael Francis (1989), *K-theory*, *Advanced Book Classics* (2nd ed.), Addison-Wesley, ISBN 978-0-201-09394-0, MR 1043170
11. Bourbaki, Nicolas (1998), *Elements of Mathematics : Algebra I Chapters 1-3*, Berlin, New York: Springer-Verlag, ISBN 978-3-540-64243-5

12. Bourbaki, Nicolas (1989), *General Topology. Chapters 1-4*, Berlin, New York: Springer-Verlag, ISBN 978-3-540-64241-1
13. Coxeter, Harold Scott MacDonald (1987), *Projective Geometry (2nd ed.)*, Berlin, New York: Springer-Verlag, ISBN 978-0-387-96532-1
14. Eisenberg, Murray; Guy, Robert (1979), "A proof of the hairy ball theorem", *The American Mathematical Monthly*, 86 (7): 572–574, doi:10.2307/2320587, JSTOR 2320587
15. Eisenbud, David (1995), *Commutative algebra, Graduate Texts in Mathematics*, 150, Berlin, New York: Springer-Verlag, ISBN 978-0-387-94269-8, MR 1322960
16. Goldrei, Derek (1996), *Classic Set Theory: A guided independent study (1st ed.)*, London: Chapman and Hall, ISBN 978-0-412-60610-6
17. Griffiths, David J. (1995), *Introduction to Quantum Mechanics*, Upper Saddle River, NJ: Prentice Hall, ISBN 978-0-13-124405-4
18. Halmos, Paul R. (1974), *Finite-dimensional vector spaces*, Berlin, New York: Springer-Verlag, ISBN 978-0-387-90093-3
19. Halpern, James D. (Jun 1966), "Bases in Vector Spaces and the Axiom of Choice", *Proceedings of the American Mathematical Society*, 17 (3): 670–673, doi:10.2307/2035388, JSTOR 2035388
20. Hughes-Hallett, Deborah; McCallum, William G.; Gleason, Andrew M. (2013), *Calculus : Single and Multivariable (6 ed.)*, John Wiley & Sons, ISBN 978-0470-88861-2
21. Husemoller, Dale (1994), *Fibre Bundles (3rd ed.)*, Berlin, New York: Springer-Verlag, ISBN 978-0-387-94087-8
22. Jost, Jürgen (2005), *Riemannian Geometry and Geometric Analysis (4th ed.)*, Berlin, New York: Springer-Verlag, ISBN 978-3-540-25907-7
23. Kreyszig, Erwin (1991), *Differential geometry*, New York: Dover Publications, pp. xiv+352, ISBN 978-0-486-66721-8
24. Kreyszig, Erwin (1999), *Advanced Engineering Mathematics (8th ed.)*, New York: John Wiley & Sons, ISBN 978-0-471-15496-9
25. Luenberger, David (1997), *Optimization by vector space methods*, New York: John Wiley & Sons, ISBN 978-0-471-18117-0
26. Mac Lane, Saunders (1998), *Categories for the Working Mathematician (2nd ed.)*, Berlin, New York: Springer-Verlag, ISBN 978-0-387-98403-2
27. Misner, Charles W.; Thorne, Kip; Wheeler, John Archibald (1973), *Gravitation*, W. H. Freeman, ISBN 978-0-7167-0344-0
28. Naber, Gregory L. (2003), *The geometry of Minkowski spacetime*, New York: Dover Publications, ISBN 978-0-486-43235-9, MR 2044239
29. Schönhage, A.; Strassen, Volker (1971), "Schnelle Multiplikation großer Zahlen (Fast multiplication of big numbers)", *Computing (in German)*, 7 (3–4): 281–292, doi:10.1007/bf02242355, ISSN 0010-485X

30. Spivak, Michael (1999), A Comprehensive Introduction to Differential Geometry (Volume Two), Houston, TX: Publish or Perish
31. Stewart, Ian (1975), Galois Theory, Chapman and Hall Mathematics Series, London: Chapman and Hall, ISBN 978-0-412-10800-6
32. Varadarajan, V. S. (1974), Lie groups, Lie algebras, and their representations, Prentice Hall, ISBN 978-0-13-535732-3
33. Wallace, G.K. (Feb 1992), "The JPEG still picture compression standard" (PDF), IEEE Transactions on Consumer Electronics, 38 (1): xviii–xxxiv, CiteSeerX 10.1.1.318.4292, doi:10.1109/30.125072, ISSN 0098-3063, archived from the original (PDF) on 2007-01-13, retrieved 2014-10-25
34. Weibel, Charles A. (1994). An introduction to homological algebra. Cambridge Studies in Advanced Mathematics. 38. Cambridge University Press. ISBN 978-0-521-55987-4. MR 1269324. OCLC 36131259.
35. Vincenzo De Risi (31 January 2014). Mathematizing Space: The Objects of Geometry from Antiquity to the Early Modern Age. Birkhäuser. pp. 1–. ISBN 978-3-319-12102-4.
36. (Boyer 1991, "Ionia and the Pythagoreans" p. 43)
37. Martin J. Turner; Jonathan M. Blackledge; Patrick R. Andrews (1998). Fractal geometry in digital imaging. Academic Press. p. 1. ISBN 0-12-703970-8. Archived from the original on 6 September 2014.