
A Survey of Default Risk Methodologies for Equity Pricing with a Focus on Advances in the Field of Default Risk Assessment and Equity Pricing in Pakistan

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Abstract: The aim of this study is to review the default risk modelling literature from the perspective of Pakistan, where an effort has been put to cover the three classes of such models i.e. Accounting, Structural and Logit based hazard models. As per review it is very clear that the first two modelling techniques are extensively employed in Pakistan with Z-Score and KMV Merton model as commonly used estimation technique. Hazard models are the least used due to data issues, as for majority of delisted firms' data is not available in data services like Thomson Reuters Eikon and Bloomberg before 2000 and even local data sources like Pakistan Stock Exchange is not of much help. Due to the reason Hazard Models are least employed for default risk estimation in Pakistan along with neural network models being most neglected one with only one paper in 2020. The review helps in identifying the gaps in this area of study, which include identification of models for future research and data issues in Pakistan along with equity pricing implications.

Keywords: Default risk methodologies, Equity Pricing, Default risk assessment

INTRODUCTION

The origin of risk management in general is traced to ground-breaking work of Bachelier (1900), where he captured the fluctuations in financial assets using Brownian Motion and introduced it as a measure of risk (Dionne, 2013). Informal risk assessment after Bachelier (1900) is traced in Markowitz (1952) and contemporary researcher's work to capture risk in terms of volatility in returns; later resulting in the development of modern portfolio theory based on Capital Asset Pricing Model (CAPM) (Sharpe, 1964), explicitly identifying risk in terms of market risk measured through Beta. Prior to their work, no citation of risk in empirical research is traced, let alone default risk was never a point of concern in contemporary research during the era; such as Altman (1968) work used ratio analysis as a tool to assess and predict financial health, taking default as an event or case for this assessment. Later Black & Scholes (1973), using the Markowitz (1952) framework introduced the revolutionary model to price options and within same year Merton (1973) extended their work to present the first default risk model. This led to offering of the extended risk coverage instruments like interest rate and currency swaps, which also lead to the introduction of the newer statistical tools for screening the customers i.e. credit scoring for credit risk and credit risk management, facilitating the risk pricing based on credit risk and in turn default risk (Dionne, 2013).

The interest in study of default risk is, therefore, many folds and includes investor's interest in higher equity premium for the default risk, arising out of overall increase in financial risk, as reported by Campbell, Hilscher, & Szilagyi (2008); apart from creditors and regulators focus on maintaining discipline in financial market by avoiding credit risk and credit institutions failure.

Empirical literature on default risk, especially since last three decades, contains the evidence related to impact of default risk and relationship with risk premia in various forms; with the change in focus of default from the perspective of debt providers and debt insurers to equity investors. Earlier studies on the subject catered to the need in terms of measurement and assessment of risk for creditors (J. Chen & Hill, 2013: 5118), as default risk assessment serves as better pricing tool for creditors and efficiently mitigate the risk (Agarwal and Taffler, 2008). The change of focus towards the relationship between equity and default risk widened the scope from credit assessment perspective to equity investor's asset pricing concerns. With this change, the evidence reported however was anomalous and casts some doubts over the pricing implications from equity investor's perspective, in related empirical work covering different models. The findings in these studies range from highest returns on high default risk (Vassalou and Xing, 2004) to low returns on high default risk equities as compared to low default risk equities (Garlappi and Yan, 2011). Vassalou & Xing (2004) in his work terms default risk as systematic risk, as they argue that it's because of this reason default risk is present in equity premia. Due to the systematic nature and implication of default risk, estimation and prediction of default risk is a core area of interest among

academicians and practitioners, as the ability to early detect the default risk has important managerial implications for firms from corporate, investment, credit and customer credit perspective, with understanding of such risk based on the statistical and intelligent models.

This manuscript is thus divided into three sections i.e. first section subdivided into three subsections covering three types of default risk models i.e. accounting based, structural default risk models and logit models popularly known as hazard models, review of equity risk premia and finally conclusion based on review of such models. This review help understand the scope of work carried over default risk modelling in Pakistan and identifies some areas of further investigation as this country lacks serious work on recent advances in this area due to data availability issues.

REVIEW OF DEFAULT PREDICTION MODELS

A number of research models have been used, where efforts have been made to explicitly or implicitly capture the default risk and understand the implications for equity pricing. Broadly these models are classified into three approaches i.e. accounting based models, structural default models and reduced form hazard models. Following is the discussion on each type and studies covering such models.

Accounting Based Models

The accounting based measure have been used since turn of this century to assess the credit worthiness of the business entities i.e. Current Ratio as ability to meet current obligations by William M. (1908). However, first notable study on the subject matter was by Beaver (1966) to verify the usefulness of the ratio analysis for decision making by the stakeholders and the firm default was used as an event to assess the same. This study used a sample of failed and non-failed firms from Moody's Industrial Manual while the selection of non-failed firms was based on paired-sample design, to control for the factors that might distort the relationship between ratios and failed firm. The analysis in this study was based on dichotomous classification test and likelihood test to predict failure on the basis of accounting data rather than financial ratios as described by Beaver (1966).

Altman (1968) work is next in line of notable works in accounting models, where the focus was efficacy of ratio analysis in the backdrop of default risk following Beaver (1966). The motivation behind his work was to use the ratio analysis as supplementary tool in comparison to statistical tools, as the contemporary researchers believed in using statistical tools as the efficient, rigorous and scientific method of research enquiry. Tracing the quantitative research studies on business failures, he used the initial studies like Merwin (1942), to identify the variables for his work. He, however, questioned the validity of the accounting ratio's for generalization on the theoretical and practical reasons and to control, used multiple discriminant analysis (MDA) to identify groups of ratios for assessing bankruptcy and assigning weights to ratios from importance and usefulness perspective. His model's outcome widely known as 'Z-Score' is based on linear relationship found among ratios identified through MDA analysis, as most appropriate ratios describing the liquidity, profitability, leverage, solvency and activities in sample cases. Despite its results, work is criticised due to issues related to MDA Analysis, whose output is a score with a very weak intuitive understanding, the sampling criteria being subjective, ageing issue as industry and market dynamics change over time, causing Z-Score re-estimation every time. Research design is also an issue as small and equal size sample was not representative of the population, with only justification of methodology being the purpose of using this technique in this study i.e. to discriminate variable or ratios in a group that may have otherwise significant relationship with financial distress but the exact importance may not be known. After this, a number of other endeavours were undertaken by Altman (1973), Altman & Loris (1976), Altman & McGough (1974) and Altman, Haldeman, & Narayanan (1977), which resulted in an improvement in originally proposed Z-Score Model. The Altman, Haldeman, & Narayanan (1977) revised output was known as Zeta Analysis, based on revision in reporting standards since 1960's resulting in accuracy for up to 5 years prior to bankruptcy.

Considering the problems associated with MDA, Ohlson (1980) used maximum likelihood estimation based on conditional logit model. This model improved the predictability to 90% as compared to Z-Score's predictability of 70%. Later studies also confirm the similar findings like Wu, Gaunt, & Gray (2010) tested the Altman (1968) and Ohlson (1980) to confirm Altman (1968) model performs poorly as compared to Ohlson model on the basis of the Receiver Operating Characteristic (RoC), a test to assess the efficacy of the two models.

Next in list of major empirical works is Zmijewski (1984), which used accounting ratios to test the procedural problems in predicting the default or financial distress of the firms and Dichev (1998) work based on finding the distress risk factor as a pricing factors in presence of size and book-to-market factors. Using Altman (1968) Z-Score and Ohlson (1980) O-score as a measure of the default risk, he analysed the relationship employing portfolio technique by constructing decile portfolios and the regression. Dichev reports portfolios having high default risk earning unusually low returns, against the traditional risk returns relationship. Dichev (1998) suggests mispricing in the market as a reason of the low returns in the firms with high default risk. Using similar sample for the period 1965-1996, Griffin & Lemmon (2002) reports similar results with Ohlson (1980) O-Score. His study is pioneering work using accounting models to associate the risk premia with default risk.

In Pakistan, the default risk and equity pricing seems to be a much neglected area, and in the context of Pakistan most of research is based on Z-score as a measure of default i.e. Rashid (2011) propose their own Z-Score Model for Pakistan, Shahzad Ijaz *et al.* (2013), Malik *et al.* (2013), who also report the default risk to be systematic risk at Karachi stock exchange. Chhapra *et al.* (2020) also have used Ohlson (1980) O-Score to estimate default and employing portfolio analysis with Capital Assets Pricing Model along with Fama French methodology to assess the pricing implications in Karachi Stock Exchange. Results are found to in line with risk return theory, as firms with default risk were found to be exhibiting higher return and among fewer works in Pakistan using accounting model for equity risk premia. Akhtar *et al.* (2018) also used Z-score to determine the factors affecting insolvency but no effort was made to redefine default risk as such.

Regardless of extensive work in accounting models since 1970's internationally and in Pakistan, the use of the accounting models for estimating default risk are criticised for not being robust, due to numerous reasons ranging from data used to research design, which makes the models handicapped in predicting the default as identified by Agarwal and Taffler (2007).

Structural Default Models

Due to issues related to accounting models the researchers started focusing on alternate models, with introduction of structural models as an option. These models see default as an event occurring when asset value reach a level which is below its outstanding liabilities, thus not sufficiently covering the creditor's claims. They follow firm value approach of Black & Scholes (1973) model, which as a part of his work on option pricing on these lines, using market data of assets and liabilities rather than accounting data for default prediction. As such models are categorised as market model or structural models as they plainly model market dynamics of assets values. Following Black and Scholes (1973) these models require strong assumptions as to the firm's assets, its debt and capital structure.

On the theoretical basis provided by Black & Scholes (1973), Merton (1973) furthered the idea of measuring the default probabilities using market based data. Black, Scholes and Merton (BSM) model directly applies the European options valuation methodology for pricing the options. While modelling the option valuation, this model made a trade-off between realistic assumptions to portray real world market along with assets trading scenario and ease of implementing the models, which would not have been possible if realistic assumptions were incorporated (Elizalde, 2005). With all the unrealistic assumptions to keep the model simple, it was found to be better predictor as compared to contemporary accounting models. It is also considered best at connecting default risk with latent structural variables with instinctive economic interpretations and an endogenous reasoning of default risk. Despite all its short comings arising out of its complicity it is considered as a benchmark for evaluating alternate models for pricing. Later on (Merton, 1973a) himself, introduced the relaxed model for American options. Later Black and Cox (1975) improved on the Merton model's assumption of option being exercised at time of maturity by incorporating the option to be exercised at the time when firm's asset values decrease below liabilities. To incorporate the improvement, Black and Cox has assumed that the bond holders have the call option, to exercise the covenant is put into bond indentures, which allows for liquidation of firms, if assets value fall below pre-determined value. The main problems with this model, even after adjusting for maturity, is still fails to allow for a debt security with coupon bonds and predictable maturity or default.

The structural modelling, thereafter, was followed by Time-changed Brownian Motion models and Kealhofer, McQuown and Vasicek (KMV) Model developed by KMV Corporation in late 80's. It is also a structural model, based on market values of publically traded companies and uses Expected Default Frequency (EDF) as a measure of implicit risk of default. Major studies based on this model were Bharath and Shumway (2004) with hazard model out performing the KMV-Merton Model, and Duan, Gauthier and Simonato (2005) which establishes superiority of Maximum Likelihood Method over KMV due to its silence over distribution properties of model estimates, however, despite the issues raised this model tends to be among popular commercial model after its acquisition by Moody's credit rating agency. Research enquiry following structural models is also limited in Pakistan is limited to Merton and KMV-Mertom models and include work by Elahi *et al.* (2014) to apply KMV model with macroeconomic dynamics in Pakistan, while no incidence of Black and Cox (1975) is available in the context of default risk studies. Ehsan Khan, Iqbal and Faizan Iftikhar (2020) work is latest in line using dynamic probit panel model, hybrid artificial neural network model based on apart from (Merton, 1973b) as a basis of using simplified KMV Merton Model to test bankruptcy estimations. They also used a hybrid artificial neural network model by combining first two models and demonstrated its superiority on all three models individually.

Hazard Models

These models are class of survival models used in statistics, to analyse data where predicted variable is length of time to the occurrence of an event, while for some sample units the event has not occurred yet and some variables have predictive effect on the time to the event that is to be assessed. In finance, the hazard models are used because of their ability to measure the bankruptcy event at each point in time, using all the information available about a firm (Lancaster, 1990). As compared to accounting and structural model i.e. 'Static Models', using only one set of data i.e. a year before bankruptcy, hazard model account for time varying characteristics of firm. In these models the dependent variable is the time, a firm is considered healthy, till it departs this sample of healthy firms.

If it is for other than bankruptcy, the firms are censored, as of no longer observed by Hazard Models, as against the static models, where they are considered healthy firms. Therefore, as per this modelling technique, the default risk changes over time during the life of a firm, while the health or the default, is based on the function measured through current data of financial ratios and its life.

Shumway (2001) is considered the pioneer in hazard models for predicting default risk, using both accounting and market data, thus combining the covariates used in preceding accounting and structural models to forecast default probabilities. The main motivation of this study was based on the fact that previous models were based on single period classification of models for default, which he terms as a static models, incorporating multi-period data. He argues that the firm change over time and approximation of the default event and related risk over its past data results in default risk measures to be biased and inconsistent, as the default event is a shock that occurs not very often, while data to estimate the even and related risk is based at least in decade preceding the event. They have used the hazard model based on logit function for predicting bankruptcy. They report the accounting variables were found to be weak predictors of firm default as compared to market driven variable and provide a better out-of-sample forecast apart from the statistical inferences produces by them are widely different from the static models. Shumway (2001) considers most of the notable static models such as Edward I Altman (1968); E.I. Altman et al. (1977); Ohlson (1980), Zmijewski (1984) as being mis-specified in terms of models, samples and data.

Chava and Jarrow (2004) used Shumway (2001), Altman (1968) and Zmijewski (1984) approach to test the default prediction efficacy of hazard Model. A further addition of industry affect was introduced to see it's impact. They claim their model to be superior due to a better sample and the superior prediction ability of the hazard rate default models using Shumway as benchmark, to compare with Edward I Altman (1968) and Zmijewski (1984) model for their sample firms and sample period. They observe that the accounting covariates add little to the predictive power of the model and for the reason hazard models outperform the two models, with hazard model predicting 74.4 percent of the defaults, while Altman's model predicting 63.2 percent and Zmijewski was only able to estimate 43.2 percent. Misclassification of firms was also found to be lowest in the hazard model as compared to the two models.

Next important study is by Campbell et al. (2008), which again used Shumway (2001) and Chava and Jarrow (2004) to measure default probabilities and use portfolios sorted on default probabilities to measure average returns and relate portfolios thus formed with size and value effects by three sorts in portfolios. Their study is set in the backdrop of higher probability of failure of some firms and the risk premium associated as such not captured by the standard CAPM model. They demonstrated that the size and value anomalies are not related to default risk; as distressed stocks had high market beta's and higher loading on size and value factors in Fama and French (1996) asset pricing model but low returns, suggesting the mispricing of default risk by market.

Jacobson, Lindé, & Roszbach (2013) used a hazard model consisting of firm specific variables and macroeconomic variables in Campbell et al. (2008) model but excluded the market variables used by the study. They have identified the relationship between macro-economic variables and default risk and report surprisingly robust explanation of default during Swedish banking crises and economic fluctuations.

Bauer & Agarwal (2014) work on default risk is also inspired by the conflicting results reported in empirical work on prediction ability of these models. They document that the existing empirical work revolves around typical comparison of the models along one of the three dimensions and document contradictory evidence on the effectiveness of contending approaches for estimating firm bankruptcy using information content tests to demonstrate that the structural approach is superior to the traditional accounting-based approach while Reisz & Perlich (2007) report opposite conclusions using Receiver Operating Characteristics (ROC) analysis. Similarly, Keenan & Sobehart (1999) document the superiority of contingent claims model over the hazard models using ROC analysis and information entropy tests but Campbell et al. (2008) use the information content test to conclude the opposite.

As far as empirical literature in Pakistan is concerned in terms of logit or hazard models, we find work still in progress and limited work in recognized journals can be cited, with notable studies by Jaffari (2017), Qayyum and Suh (2019) and Ehsan et al. (2020) with most of studies focused on measurement of default risk and its determinants and only few covering pricing and anomalous results of models.

EQUITY RISK RETURN RELATIONSHIP USING DEFAULT RISK:

A number of studies have related the default risk as a pricing factor in asset pricing models, with recognition of default risk as one of the anomaly and majority of these studies used the extensions of Merton (1973) model. Vassalou & Xing (2004) work is among the pioneering study, to measure default risk using Default Likelihood Indicator (DLI). While measuring and relating the default risk with equity returns, they also have tested the book-to-Market and Size effect being related to the default risk, as reported in by E. Altman (1984) and Favara et al. (2012). They argue that the small firms returns are greater than big size firms, if default risk is high, as an evidence of the classical risk return relationship, subject to the condition that firms are either small in size or had high book-to-market ratio. Vassalou & Xing (2004) also reported the evidence of default risk being systematic risk using the

asset pricing tests of Fama & French (1996), in their three factor model. They report, the evidence of some information in these firm characteristics affecting prices but they don't find evidence of the size and book-to-market factors as related to default risk as such.

Following the methodology of Vassalou & Xing (2004), Da & Gao (2010) also worked on this puzzle using similar sample of data and same time period but using the next six months returns rather than next one month returns. For them, the relationship between default risk and equity returns is positive as documented by Vassalou & Xing (2004) but only in the first month when default risk shock is faced by the firm and after the default risk shock, the gap between the high default risk firms and lower default risk firms return is reduced to insignificant levels. For them the firms with higher default risk face the change in clientele, as the institutional investors drive away their investments to better and safe firms for their focus on quality investments for their stockholders. They, therefore, related their and Vassalou & Xing (2004) findings with liquidity shock phenomenon.

Campbell, Hilscher and Szilagyi (2005) explored the determinants of corporate failure in their work. They found that firms with lower past stock returns, more volatile past stock returns and lower prices per share among other some other factors were more likely to default. They used calculated probability of default from this model to test the pricing of financially distressed stocks. Another pioneering study, using multiple default risk models including Merton (1973) methodology by Chava & Purnandam (2010) has worked on relationship between equity returns and default risk to test the pricing effect of default risk. They were able to report different results as they found reliable evidence of underperformance of distressed stocks as reported by Campbell, Hilscher, & Szilagui (2008) with significant and positive relationship between default risk and equity returns. Sudheer & Amiyatosh (2010) work on default risk as an anomaly have used two models of default risk to measure the default to compare the performance and effect on equity premiums, against the evidence reported by a number of empirical works showing negative relationship against the popular and classical evidence of positive relationship between the risk and return. They while using Merton (1973), Shumway (2001), Chava & Jarrow (2004) and Campbell et al. (2008) methodology to capture the default risk, argue that the negative relationship may be reflecting market inefficiency. Quoting Elton (1999) they argue that the small sample periods fail to capture the default or any other event, significantly carrying pricing information, as they may fail to neutralize each other. While Lundblad (2006) provides the evidence of extensive and long period samples of realized returns, for capturing the positive relationship between risk and return. They believe that the problem is acute by design in default based portfolio samples where average stock is highly likely to default, while positive portfolio return would only be observed if some of the stocks earn extreme positive returns. As per Sudheer & Amiyatosh (2010) the results as such are not related to stock anomaly rather to the low realized returns during the period.

Garlappi & Yan (2011) in their empirical endeavour have tried to test the default risk and equity returns using KMV model as suggested by Bharath & Shumway (2004), which is extension of Merton (1973). Their main focus is on anomalous pattern of earning reported in the presence of default risk factor in the empirical literature. They report the equity beta and stock returns to have a hump shaped relationship with default probabilities and this explains the negative relationship between the stock returns and default risk as reported by Campbell et al. (2008) and George & Hwang (2010). The results were found to be robust with a refined version of Jegadeesh & Sheridan (1993) momentum profits.

CONCLUSION

The details of literature covered reveal that the an extensive research has been carried over since 1966 till to date resulting in a number of models predicting default, especially after 2008 global financial crises, triggered by the collapse of US financial markets and mortgage backed securities. However, in the context of Pakistan most of research is based on Z-score as a measure of default i.e. Rashid and Abbas (2011), Ijaz et. al. (2013), Malik et. al. (2013) and Rashid and Abbas (2011) estimating their own Z-Score for Pakistan. Therefore, a wide gap has been identified as to using sophisticated approach such as Hazard Model suggested by Campbell *et. al.* (2008), and only one study by Elahi, Hussain Awan and Mehmood (2014) incorporating macroeconomic factors with Moody's KMV in comparison to Jacobson et. al. (2011) intuition of supplementing the hazard model with macroeconomic factors. While the existing literature like Chhapra *et al.* (2020) have tested the CAPM model by Sharpe (1964), Lintner (1965), apart from Fama & French (1993) three factor and Fama and French (2014) model with risk based portfolio. But no study has been based on the default risk augmented in the different assets pricing models as pricing factor. Apart from this methodological deficiency, the review identifies macroeconomic factors and corporate governance is among factors empirically been related to success and efficiency of listed firms but has not been incorporated as pricing factor, thus the gap is wide in even testing models like Campbell *et al.* (2011), let alone newer models based on neural networks were completely missing in academic research with first study of its kind published in 2020 by Ehsan Khan, Iqbal and Faizan Iftikhar, (2020) who augmented neural network modelling with dynamic Panel probit to construct the default risk factor for testing Fama and French. The lack of such deficiency seems to be due to availability of firm related accounting data in general and market data in particular, as data seems to be missing for most of delisted companies prior to 2000 and even after 2000 same issue is observed for delisted companies. With the level of missing data, a robust research enquiry seems to be

difficult with hazard models in particular in Pakistan and results in unpublished work have found to be meaningless default probabilities.

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