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Stages of Learning and Progression in Mathematics

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Abstract
There is not a country in the world which functions without supplying its schools with teaching content for every grade. This kind of syllabus is called “curriculum”. It is essential to define grade expectations which should fulfill three chief objectives: pinpointing important teaching content, characterizing learning progression which, if properly implemented, will enable students to get ready for professional life and coordinating various mathematical topics. Teachers face a huge variety of learning methods for their students while choosing concrete topics for different grade levels.

There can bedifferent (either low or high) expectations for grade levels of student preparation. Efficient teaching should always comply with learner’s intellectual abilities. Thus, implicit contradiction between meeting grade level expectations and satisfying the personal requirements of students in inevitable. The connection between knowledge and possible grade expectations leads to certain dilemma which includes both subjective and objective points of view. The general aims of the Education 2030 project are about solving this discrepancy.
Keywords: GlobalLab, grade levels, artificial intelligence, mathematical model, educational pathway, GNU Octave.

I. INTRODUCTION

There are different ways of facing problems for different disciplines: employing scientific methods, mathematical modelling, literature review, historical analysis etc. Speaking about the physics of protons, the mathematical Cascase-Probalistic method is widely used [29]. Design of scientific programs has a crucial meaning forresearchers since these programs should be built on facts. If students can implement these research methods in their learning lifestyles, it will boost their educational proficiency. This tendency has recently become especially vital as the virtual reality in the educational system has drastically gained its tempo [30]. The effective curriculum organization gives way to opening to another creative ideas and simplifying the process of learning as much as [25, 27]. Besides, this could aid in developing education in both pragmatic and scientific domains [31]. Moreover, it enables successful integration in the society and, thus, it helps students to maintain and balance their semantic unity [28]. Any kind of pedagogical learning can have a beneficial impact on the educational outcomes. Besides, it can create personal educational pathways owing to part or whole automatization and increase in the learning value. [1, 12, 16, 24, 26].

Learning pathway in mathematics

Educational pathway concepts trace back to the times of starting thedevelopmental psychology which postulated kids as adults. These grown- ups possess their own world vision which can drastically differ from the general one but which is really very creative[14, 15]. As a result, the scholars have tried to explore the interconnection between children and adults’ methods of learning. Investigation by Piaget demonstrates this phenomenon [17-19]. This finding is extremely vital for educators who believe that “you have to start from where the student is”. According to Simon’sconstructivist point of view (1995),the teacher can help students to develop more complex thinking in order to reach a target. Moreover, he introduces a special theory called a “hypothetical learning trajectory” (HLT) which involves learning actions, thinking and purpose [13, 20-22]. Thus, essentialendeavors have been carried out with the aim to shape student thinking.

Artificial intelligence methods that are implemented for the elaborating specific ways of

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automatic control canlead to a positive educational outcome [6, 23]. Thus, firm urges for teachers to put these efforts into realityare always welcome. However, each teacher is not expected to redesign totally a brand new approach in teaching. Learning pathways are mainly about informing teachers about which direction to take in teaching. Learning improvements trace key pointsenable students to development at (Corcoran, Mosher, &Rogat, 2009). All these factors stimulate valuable appreciations. In mathematics they are called learning trajectories that involve certain presumptions about student improvements and phenomena that might back them up in gradually advancing toward the objectives of mathematics at school. Many investigators have produced their learning stylessuch as counting or multiplicative thinking. With many similarities, there are also some discrepancies among scholars on these learning strategies. Thus, the consolidation is needed. Integration would also give knowledge on learning trajectories and, in such a way, help curriculum organizers to define what topics are most burning for a proper student understanding of mathematics.

II. METHODOLOGY

The researches will try to conduct different examinations in order to forecast educational pros and cons and their possible modernizations. Namely, they try to introduce innovative interactive models of education. The training sessions will provide students with practical preparation for possible implementation of the knowledge obtained. Unanimous efforts equal to exchange of ideas among learners. Besides, modeling of the cognitive processes is essential for full development of students with better performance. It has been claimed that learning trajectories enable knowledge sharing through patterns of thinking, giving mutual feedbacks and improving the strategy for inclusiveness.

However, such an implementation of trajectories can give the impression that they are aimed atcatching the true nature of learning process. Such an impression is illusionary. The grain of truth lies in the fact that children's attention to numbers from the very young age can improve in really unpredictable directions. However, it has been proved that age factor causes versatilities in opinions. Some cultural contexts show that differences in worldview, language and culture cause definite responses in the learning process. They also contribute to successful understanding and interpretation of the knowledge gained.

There are, though, many gaps in the field of computer programs for modelling applications. What is generally safeguarded is not a definite model but the method of its implementation.

The actions performed through mathematical models in the learning process are in the following order:

1. The list was created with the students' marks. Then, all the actions of learners were divided into 2 groups. The first one involved step-by-step procedure and the second one covered indicators of success or failure.

2. The accuracy of the right order of the events was increased by extra educational steps [1, 7-11].

III. RESULT AND DISCUSSION

The kind of events that characterize the trajectory of the Table 1.

Table 1. List of event types considered due to the mathematical model of the educational trajectory

No.	Event type
Group 1	
1	The user recognizes the project.
2	The user confirms the idea.
3	User approves blog post.

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4	User confirms comment.
5	The user logged in to the system.
6	The user has been activated.
7	The user has been registered.
8	The user has logged out.
9	The user creates a project.
10	The user creates an idea.
11	The user creates a school or group.
12	The user creates a blog post.
13	User comments on the project (in discussion).
14	The user comments on the idea.
15	The user comments on the blog post (news or project).
16	The user sends the project for moderation.
17	The user saves the questionnaire in the project (does not send).
18	User edits his profile.
19	User edits the project.
20	User is editing a group or school.
21	User edits portfolio.
22	User edits an idea.
23	The user sends the LAN to the user.
24	A user sends an invitation to another user.
Group 2	
25	User has been tested for GlobalLab (success/failure)
26	The user received a rating on the subject (success/failure)
27	The user completed the task as part of the GlobalLab course (success)
28	The user received an award (success).
29	The user sends the questionnaire to the project (success).
30	The user received a certificate (success).
31	The user's project was not approved by the moderator (failure).
32	The user profile was blocked by the moderator (failure).
33	User profile blocked by moderator (failure).

All the before mentioned steps of interest implementation are carried out disregarding their communal environment. While these interests get more interconnected between specific personalities, they get more self-maintaining. Among the opportunities to develop personally are initiating own project, couching or becoming more technology-oriented [27].

A decision-making process is performed through a definite methodology. The learner is observed and recorded according to his/her performance. Moreover, the student is obliged to act independently [2, 3, 5].

Learning process at this step demands two levels of actions.

Stage I. Simultaneous rehearsal.

Stage II. Setting motivational objectives in the process of jointly organized teaching and learning activities.

At this step, the student opts for a job which can define him/her as a separate personality. The specific issues are fast resolved. It is important to manoeuvre the preliminary process and give the students freedom of actions. As a consequence, the student can offer a system of decision making tools.

1. *Come up with a n individual perception of time*

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The student looks through the work, provides a series of mathematical problems, then he separates their elements(academic units). It gives information about perception and its initial transactions.

2. Structural information (analysis) to select from

The student is supposed to master the teacher materials using complexity methodology. For example, one can suggest the following chain level:

knowledge acquiring, sharing, newer problem, creative application.

3. Analysis of proposed tasks

At this step, the student always selects only important results.

4. Prioritizing activities

The juxtaposition of chosen elements of student's skills and gained knowledge. At this point the student may reject some alternatives because there is evident contradiction between the knowledge and competencies needed.

5. Embody into reality own initiatives

6. Fulfilling the tasks (with internalization method)

Students embark on problem shooting. It can happen that the task is too easy or too difficult. Thus, the task can be redefined. At the same time, the learner should identify the reason why he or she made a mistake in their decision.

7. Identifying issues in various activities

During problem shooting students can face difficulties caused by unawareness, inattention, discrepancy between theory and practice. At the next stage, students intuitively underline new issues and their reasons. 8. Problem clarification

By choosing a certain task, the student gets specific information about the preparation of the learning process and then he selects the way to study: conscious choice enables him to be always motivated.

Operating. At this level, the student resolves the problem of acting. Solutions are in the developmental stage in order to navigate situations.

The student learning process at this stage occupies two stages of student activity.

Stage III Researching, analyzing, reviewing information by students

Stage IV Interpreting information and inventing a new course of action.

Organizational. At this stage, the student clarifies the structure of the learning mode. He includes it into the system of the already existing knowledge.

Stage V. Including a new mode of action into the knowledge system.

Stage VI Students assess their activities [4].

IV. CONCLUSION

Learning trajectories can be applied both in the professional development and teaching in mathematics. For example, some mathematical teachers already defined themselves not as passing through a curriculum but as helping students to navigate specific levels of understanding. Further, researchers state that professional development concentrated on learning trajectories is beneficial for teachers' professional knowledge. Moreover, it boosts the confidence of learners. So, learning trajectories can facilitate developmentally appropriate teaching and learning for all students.

Learning trajectories may cause improvements in the design of teacher education programs as well as of programs provided by local and regional providers. For example, such programs could help teachers manage deeper understanding of the central ideas in mathematics, the way students typically manage these ideas and develop more elaborated understanding over time. Trajectories could be helpful in detecting how student understanding develops over time.

The final list of events is made and it is reviewed by the mathematical trajectory. This list enables us to include it in a wide range of educational activities.

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REFERENCES

1. T. V. Krupa, *Theoretical studies of the performed for the Stages of the ASR tasks*. Moscow: GlobalLab, LLC, 2018.
2. L. Furukawa, "Trajectory of Learning Experience based on the Performance of Canada's Youth in Mathematics", *International Journal of Innovation in Science and Mathematics Education, 26*(6), 62–75, 2018.
3. G. Lipatnikova, A. S. Polyanina, "The formation of the target component of educational activities of students using decision-making techniques in the framework of the reflexive approach", *Problems and Methods of Teaching Natural Sciences and Mathematics: Materials of the III All-Russian Scientific and Practical Conference (Ekaterinburg, December 2007)* (pp. 194-197). Ekaterinburg: Publishing House of the Ural Institute of Economics, Management and Law, 2007.
4. G. Lipatnikova, "The creation on an individual educational thakectory as one of ways to reach students in decision making", *Fundamental researches,* 5: 108-110, 2009.
5. O. A. Granichina, "Mathematical models of the quality control of the educational process in the university with active optimization", *Stokhaisticheskayaoptimizatsiya v informatike,* 2: 77-108, 2006. Retrieved from https://www.math.spbu.ru/user/gran/sb2/granolga.pdf
6. M. Taguma, F. Gabriel, M. H. Lim, "Future of Education and Skills 2030: Curriculum anaktsis: A synthesis of research on learning trajectories/Progressions in mathematics", *OECD. EDU/EDPC(2018)44/ANN3*, 2019.Retrieved from https://www.oecd.org/education/2030-project/about/documents/A_Synthesis_of_Research_on_Learning_Trajectories_Progressions_in_Mathematics.pdf
7. C. Haas, A. Hadjar, "Students' trajectories through higher education: a review of quantitative research", Higher Educarion: 1-20, 2019. DOI: 10.1007/S10734-019-00458-5
8. Anwar, I. Rofiki, "Investigatigating students' leatning trajectory: a case on triangle", *Journal of Physics Conference Series, 1088*: 012021, 2018. DOI: 10.1088/1742-6596/1088/1/012021
9. "Individual educational trajectory", *University of Tyumen,* 2019. Retrieved from https://www.utmn.ru/en/study-with-us/individual-education/
10. P. Sztajn, J. Confrey, P. H. Wilson, C. Edgington, "Learning Trajectory Based Instructions: Toward a Theory of Teaching", *Educational Researcher,* 41 (5): 147-156, 2012. Retrieved from https://www.jstor.org/stable/23254092?seq=1
11. D. Siemon, T. Barkatsas, R. Seah, *Researching and Using Progressions (Trajectories) in Mathematics Education.* Brill, 2019. ISBN: 9004396446.
12. D. B. Edwards Jr. *The Trajectory of Global Education Policy: Community-Based Management in El Salvador and the Global Reform Agenda.* Springer, 2017. ISNB: 1137508752.
13. E. Merzon, E. Galimullina, E. Ljunimova, "A smart trajectory model for teacher training", In *Cases on Smart Learning Environments* (pp. 164-187), 2019. DOI: 10.4018/978-1-5225-6136-1.ch010
14. D. Johnson, "Comparing the trajectories of educational change and policy transfer in developing countries", *Oxford Reciew of Education, 32* (5): 679-696, 2006. Retrieved from https://www.jstor.org/stable/4618689?seq=1
15. M. N. Ryzhkova, "The mathematical model of the process of educayion management", *BestnikCherepovetskogogosudarstvennogouniversiseta, 6*: 41-47, 2015. Retrieved from https://cyberleninka.ru/article/n/matematicheskaya-model-protsessa-upravleniya-obucheniem

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16. P. V. Ovchinnikov, *Mathematical Models and Instruments for Designing Adaptive Educational Trajectories for Preparation of Competitive Specialists in Universities:* Abstract thesis. Rostov-on-Don, 2014. Retrieved from http://economy-lib.com/matematicheskie-modeli-i-instrumentariy-proektirovaniya-adaptivnyh-obrazovatelnyh-traektoriy-dlya-podgotovki-konkurentosp
17. S. A. Vdovina, I. M. Kungurova, “The nature and directions of the individual educational trajectory”, *Naukovedeniye, 6*: 1-8, 2013.
18. M. Brown, “Six trajectories for Digital Technology in Higher Education”, *Educause review*, 2015. Retrieved from https://er.educause.edu/-/media/files/article-downloads/erm1541.pdf
19. “Transforming Our World, the 2030 Agenda for Sustainable Development”, *United Nations General Assembly Resolution A/RES/70/1,* 2015. Retrieved from http://www.un.org/ga/search/view_doc.asp?symbol=A/RES/70/1&Lang=E
20. A. C. Antoulas (Ed.), *Mathematical System Theory*. Berlin: Springer, 1991.
21. J. G. Maree, “Narrative Research in Career Counselling: The Career Construction Interview”, In L. Sumaya, F. Angelo, K. Sherianne, *Transforming Research Methods in the Social Sciences: Case Studies from South Africa* (pp. 186-202). Johannesburg: Wits University Press, 2019. Retrieved from www.jstor.org/stable/10.18772/22019032750.17
22. A. P. Adiredja, “Anti-Deficit Narratives: Engaging the Politics of Research on Mathematical Sense Making”, *Journal for Research in Mathematics Education, 50* (4): 401-435, 2019. Retrieved from www.jstor.org/stable/10.5951/jresematheduc.50.4.0401
23. M. McCartney, *The Indian Economy*. Newcastle upon Tyne: Agenda Publishing, 2019. DOI: 10.2307/j.ctvnjbfk1
24. R. L. Geiger, *American Higher Education since World War II: A History*. Princeton University Press, 2019. Retrieved from www.jstor.org/stable/j.ctv9hvtbz.
25. M. Ndlovu, “Modeling with Sketchpad to enrich students' concept image of the derivative in introductory calculus: developing domain specific understanding”, 2008. Retrieved from https://www.researchgate.net/publication/323120414_Modeling_with_Sketchpad_to_enrich_students'_concept_image_of_the_derivative_in_introductory_calculus_developing_domain_specific_understanding
26. L. Darling-Hammond, L. Flook, C. Cook-Harvey, B. Barron, D. Osher, “Implications for educational practice of the science of learning and development”, *Applied Developmental Science,* 2019. DOI: 10.1080/10888691.2018.1537791
27. S. Hidi, K. A. Renninger, “The four-stage model of interest development”, Educational Psychologist, 41 (2): 111-127, 2006. DOI: 10.1207/s15326985ep4102_4
28. A. Pylkin, O. Stroganova, N. Sokolova, M. Pylkina, “The Development of Information Technology and the Problem of Identity”, *Proceedings of 8th Mediterranean Conference on Embedded Computing, MECO 2019*. Budva, Montenegro, 2019. DOI: 10.1109/MECO.2019.8760159
29. T.A. Shmygaleva, A.I. Kupchishin, A.A. Kupchishin, C.A. Shafii, “Computer simulation of the energy spectra of PKA in materials irradiated by protons in the framework of the Cascade-Probabilistic method”, *IOP Conference Series: Materials Science and Engineering*, 510: 012024, 2019. DOI: 10.1088/1757-899X/510/1/012024
30. N. Ivanova, T. Sorokina, “The relationship between the categories “Educational environment” and “Educational space” in Russian psychological and pedagogical science”, *RevistaInclusiones*, 7 (Esp.): 100-118, 2020.
31. E. V. Dudukalov, A.B. Laptander, “Obrazovaniyeinauka: kakprodolzhat' modernizatsiyu v usloviyakhretsessii? [Education and science: how to continue the modernization during recession?]”, *Naukaiobrazovaniye: khozyaystvoiekonomika; predprinimatel'stvo; pravoiupravleniye*, 1: 7-13, 2015.

P-ISSN: 2204-1990; E-ISSN: 1323-6903
DOI: 10.47750/cibg.2021.27.02.518

32. Bezuk, N. S., &Bieck, M. (1993). Current research on rational numbers and common fractions: Summary and implications for teachers. In D. T. Owens (Ed.), Research ideas for the classroom: Middle grades mathematics (pp. 118-136). New York: Macmillan.
33. Barrett, J., Clements, D., Cullen, C., McCool, J., Witkowski, C., &Klanderman, D. (2009, April). Children's abstraction of iterative units to measure linear space: A trajectory. Paper presented at the Annual Meeting of the American Educational Research Association (AERA). San Diego, California.
34. Clements, D., &Sarama, J. (2007b). Early childhood mathematics learning. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (Vol. 1, pp. 461-555). New York: Information Age Publishing.
35. Freudenthal, H. (1987). Mathematics starting and staying in reality. In I. Wirszup and R. Street (Eds.), Proceedings of the USCMP Conference on Mathematics Education on Development in School Mathematics around the World, NCTM, Reston, VA.
36. Edgington, C. (2012). Teachers' uses of a learning trajectory to support attention to students' mathematical thinking. Unpublished doctoral dissertation. North Carolina State University, Raleigh NC.
37. Empson, S. B. (2011). On the idea of learning trajectories: Promises and pitfalls. The Mathematics Enthusiast, 8(3), 571-596.