Journal of Contemporary Issues in Business and Government Vol. 27, No. 2, 2021
https://cibg.org.au/
P-ISSN: 2204-1990; E-ISSN: 1323-6903
DOI: 10.47750/cibg.2021.27.02.430

Constructing of Some New Results in Intuitionistic Fuzzy Metric Space by Applying the Abstraction of Fuzzy Iterated Contraction

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Abstract: Fixed point results are established at the basis of Banach contraction principle. Many authors extended this result in different form for different type of mappings in various ways according to their applications. The extension can be useful for uncertainty theory obtained by fuzzy sets and soft sets. In this paper, we establish a couple of fixed-point results for fuzzy iterated contraction in the intuitionistic fuzzy metric space. Obtained results are generalization form of well-known results . The established results can further be generalized for soft sets as fixed point theorems for fuzzy soft sets or soft fuzzy sets. The couple fixed point theorems will be useful in the management study as well as in stock marketing.

Keywords:Fuzzy iterated contraction map, Fuzzy metric space, Intuitionistic fuzzy metric space, t-conorm, t-norm, stock marketing

INTRODUCTION AND PRELIMINARIES

Metric space has been initiated by French Mathematician Frechet [9]. It defines that an abstract set X and distance function $d: X \times X \to R$ defined on X called metric space in case it fulfill the following properties such that for x, y, z, ∈ X

1. $d(x,y) \geq 0,\ d(x,y) = 0 \Leftrightarrow x = y$
2. $d(x,y) = d(y,x)$
3. $d(x,z) \leq d(x,y) + d(y,z)$

As the entireworld is surrounded by uncertainty, the information or data we obtain from the environment is vague or incorrect. To deal with this vagueness the Iranian born scientist Zadeh [24] was instrumental in introducing the fuzzy set which is infused as "A fuzzy set A in X is a function with domain X and values in [0,1]".

Kramosil and Michalek [13] generalize the statistical metric space and it is considered a fuzzy metric space. Afterward, George and Veeramani [10] prove some known results by defining the Hausdorff topology and faintly altered the abstractionof fuzzy metric space, is defined as "If X is a random (nonempty) set, $*$ is continuous t-norm andfuzzy setd_F on $X^2 \times [0,\infty]$ fulfilling the following terms, for all $x, y, z \in X$ each t &$s > 0$"

1. $d_F(x,y,t) > 0$,
2. $d_F(x,y,t) = 0$ iff $x = y$,
3. $d_F(x,y,t) = d_F(y,x,t)$,
4. $d_F(x,y,t) * d_F(y,z,t) \leq d_F(x,z,t+s)$,
5. $d_F(x,y,\cdot): (0,\infty) \to [0,1]$is continuous,

Then the 3-tuple $(X, d_{F,} *)$ is said to be fuzzy metric space andd_F is named as fuzzy metric on X . then $d_F(x,y,t)$ denotes the nearness degree betweenx & ywith respect tot.

Atanassov [4]introduced the generalization of fuzzy set as intuitionistic fuzzy setwhich is interpreted as "letX be a non-empty set, then an intuitionistic fuzzy set A such that

$A = \{< x, (\mu_A(x), v_A(x) >: x \in X\}$

Where the function $\mu_A(x) \to [0,1]$ and $v_A(x) \to [0,1]$ symbolize the degree of membership & the degree of nonmembership of each element $x \in X$ of the set A respectively and $0 \leq \mu_A(x) + v_A(x) \leq 1$ for each$x \in X$".

Later on Atanassov (1987, 1988&1994) defined some new results and new operations on intuitionistic fuzzy sets. We refer to see [5],[6],[7] for more results on intuitionistic fuzzy sets.

Prolonged the Banach contraction theorem Grabiec [11] defined the G-complete fuzzy metric space. Çoker [8] proposed the concept of "Intuitionistic fuzzy topological spaces". It defines the belief of Intuitionistic fuzzy topological spaces such that

$0_{\sim} = \{< x, 0,1 >: x \in X\}$
$1_{\sim} = \{< x, 1,0 >: x \in X\}$

"An intuitionistic fuzzy topology in a nonempty set X is a family τ of Intuitionistic fuzzy set in X satisfy the following axioms:
$(T_1) 0_{\sim}, 1_{\sim} \in \tau$;
$(T_2) G_1 \cap G_2 \in \tau$ for any of $G_1, G_2 \in \tau$;
$(T_3) \cup G_i \in \tau$ for some arbitrary family $\{G_i: i \in J\} \subseteq \tau$
The(X, τ) is termed as Intuitionistic fuzzy topological space".
The concept of t-norm & t-conorm established by Schweizer and Sklar [18], which was very useful in introducing Intuitionistic fuzzy metric space.
t-norm: "A binary operation $*: [0,1]^2 \to [0,1]$ is a continuous triangular norm (t-norm) if it fullfills the following conditions.
1. $*$ is commutative and associative,
2. $*$ is continuous,
3. $a * 1 = a \ \forall a \in [0,1]$,
4. $a * 1 \leq c * d$whenever$a \leq c$ & $b \leq a \ \forall \ a, b, c, d \ \in [0,1]$."
t-conorm: "A binary operation $\diamond: [0,1]^2 \to [0,1]$ is a continuous triangular conorm(t-conorm) if it fullfills the following conditions:
1. $\diamond$ is commutative and associative,
2. $\diamond$ is continuous,
3. $a \diamond 0 = a \ \forall \ a \in [0,1]$,
4. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.
Fixed point theorems for fuzzy contractive mappings explored by Gregori &Sapena [12]. Then, Žikić [23]proved somefixed point theorems for mappings on fuzzy metric space which was the improved result of Gregori &Sapena.Before introducing intuitionistic fuzzy metric space Amini &Sadati [2][3]defined some new results for fuzzy metric space, defined properties of t-norm and s-norm. Park [15]commenced the concept of intuitionistic fuzzy metric spaces After that many authors studied this concept and generated fixed point theorems in Intuitionistic fuzzy metric space. Saadati, &Park [17] came up with a new notion of Intuitionistic fuzzy topological spaces.Further, we refer to see [14] [1] [19].
A 5-tuple $(X, M, N, *, \lozenge)$ is articulated be an intuitionistic fuzzy metric space(Ifms) if X is an random set,$\lozenge$ a continuous t-conorm and$*$ a continuous t-norm and and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions; $\forall \ x, y, z \in Xs, t > 0$
1. $M(x, y, t) + N(x, y, t) \leq 1$,
2. $M(x, y, 0) = 0$,
3. $M(x, y, t) = 1 \ \forall \ t > 0 \ iff \ x = y$,
4. $M(x, y, t) = M(y, x, t)$,
5. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s) \ \forall \ x, y, z \in X \ \ s, t > 0$,
6. $M(x, y, \cdot): [0, \infty) \to [0,1]$is left continuous,
7. $lim_{t \to \infty} M(x, y, t) = 1 \ \forall \ x, y \in X$,
8. $N(x, y, 0) = 1$,
9. $N(x, y, t) = 0 \ \forall \ t > 0 \ iff \ x = y$,
10. $N(x, y, t) = N(y, x, t)$,
11. $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s) \ \forall \ x, y, z \in Xs, t >0$,
12. $N(x, y, \cdot): [0, \infty) \to [0,1]$is right continuous,
13. $lim_{t \to \infty} N(x, y, t) = 0 \ \forall \ x, y \in X$.
Then (M, N) is an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$&$N(x, y, t)$denote a degree of nearness and a degree of non nearness between x&y with respect to t.
The Iterated contraction is studied by Rheinboldt [16]. The notion of iterativecontraction turns out to be very fruitful in the study of definite iterative process and has broadapplicability in metric space. With the help of this concept and fuzzy metric space, Xia & Tang [20], [21] introduced the idea of fuzzy contraction map which is induced as "If $(X, M, *)$ is fuzzy metric space such that $M(Tx, T^2x, t) \geq M(x, Tx, \frac{t}{k})$for all $x \in X, t > 0, 0 < k < 1$ then T is stated to be fuzzy iterated contraction (FIC)".
Example 1.1**:**If $T: \left[\frac{-1}{3}, \frac{1}{3}\right] \to \left[\frac{-1}{3}, \frac{1}{3}\right]$ is given by $Tx = x^2$, then T is a fuzzy iterated contraction but not a fuzzy contraction map.

MAIN RESULT

Lemma 2.1: Let $T: X \to X$ a FIC map in $(X, \mathbb{M}, \mathbb{N}, *, \lozenge)$ intuitionistic fuzzy metric space fullfills the following conditions:

1. If $x = Tx$ then $\mathbb{M}(Tx, T^2x, t) \geq \mathbb{M}(x, Tx, t)$&$\mathbb{N}(Tx', T^2x', t) \leq \mathbb{N}(x', Tx', t)$
2. If $x_{n+1} = Tx_n$ is the iterates sequence for some $x \in X$ owna convergent subsequence which converges to y&T is continuous at y then Tpossess a fixed point.

PROOF:Let the sequence $\{\mathbb{M}(x_{n+1}, x_n, t)\}$ is a sequence of real numbers which is non-decreasing, bounded above by 1 then definitely there exist a limit. As it is given that the subsequence converges to y and T is continuous on X therefore $T(x_{n_i})$ converges to Ty and $T^2(x_{n_i})$ converges toT^2y.

$\mathbb{M}(y, Ty, t) = \lim_{n\to\infty} \mathbb{M}(x_{n_i}, x_{n_{i+1}}, t) = \lim_{n\to\infty} \mathbb{M}(x_{n_{i+1}}, x_{n_{i+2}}, t) = \mathbb{M}(Ty, T^2y, t)$

Let $y \neq Ty$ then $\mathbb{M}(Ty, T^2y, t) > M(y, Ty, t)$ since T is a FIC map then by definition $\mathbb{M}(Ty, T^2y, t) = \mathbb{M}(y, Ty, t)$ But

$\mathbb{M}(Ty, T^2y, t) > M(y, Ty, t)$which is a conflict and hence $Ty = y$.

Similarly the sequence $\{\mathbb{N}(x'_{n+1}, x'_n, t)\}$ is a sequence of real numbers which is non-increasing, bounded below by 0, then there exist a limit and the subsequence converges to y' and T is continuous on X therefore $T(x'_{n_i})$converges to Ty' and $T^2(x'_{n_i})$ converges to T^2y'.

$\mathbb{N}(y', Ty', t) = \lim_{n\to\infty} \mathbb{N}(x'_{n_i}, x'_{n_{i+1}}, t) = \lim_{n\to\infty} \mathbb{N}(x'_{n_{i+1}}, x'_{n_{i+2}}, t) = \mathbb{N}(Ty', T^2y', t)$

Let $y' = Ty'$ then $\mathbb{N}(Ty', T^2y', t) < N(y', Ty', t)$ since T is FIC map then by definition $\mathbb{N}(Ty', T^2y', t) = \mathbb{N}(y', Ty', t)$ but $\mathbb{N}(Ty', T^2y', t) < N(y', Ty', t)$which is a conflict and therefore$Ty' = y'$.As we proved that T has a converging point in both the cases so we can say that Tpossess a fixed point.

Theorem 2.2:Let $T: X \to X$ is a FIC map with respect to ϕ and ψ and $(X, \mathbb{M}, \mathbb{N}, *, \lozenge)$ is a complete Ifms.

$\mathbb{M}(x, Tx, t) \leq \phi(x, Tx, t)$,

$\mathbb{N}(x, Tx, t) \geq \varphi(x, Tx, t) \forall\, x \in X, Tx \in X$

The sequence$x_n = T^n x$of iterates for $x \in X$ converges to $y \in X$ and T possess a fixed point If T is continuous at$y$.

PROOF: As it is quite obvious to show that $\{x_n\}$ is a Cauchy sequence in respect ofϕ and φBecause of

$\mathbb{M}(x, Tx, t) \leq \phi(x, Tx, t)$

$\mathbb{N}(x, Tx, t) \geq \varphi(x, Tx, t) \forall\, x \in X,\ Tx \in X$.

Consequently x_n is a Cauchy sequence with respect to $\mathbb{M}$ and$\mathbb{N}$. The sequence converges to y in $(X, \mathbb{M}, \mathbb{N})$ as it is complete the function T is continuous at y.

$y = \lim_{n\to\infty} x_n = lim f(f^{n-1}x)$

$= lim\, f x_{n-1} = f\, lim x_{n-1} = Ty \Rightarrow y = Ty$

Therefore T possess a fixed point.

Theorem 2.3: Let $T: X \longrightarrow X$ be a continuous FIC map such that if $x = Tx$ in Ifms then $\mathbb{M}(Tx, T^2x, t) > M(x, Tx, t)$, $\mathbb{N}(Tx, T^2x, t) < N(x, Tx, t)$ and the sequence $x_{n+1} = Tx_n$ own a convergent subsequence therefore $\{x_n\}$ converges to a fixed point which is of T.

PROOF: As it is also quite obvious to show that the sequence $\{\mathbb{M}(x_n, x_{n+1}, t)\}$ is a non-decreasing sequence which is bounded above by 1 and the sequence $\{x_n\}$ converging to y.

$\mathbb{M}(y, Ty, t) = \lim_{n\to\infty} \mathbb{M} = \lim_{n\to\infty} \mathbb{M}(x_{n_{i+1}}, x_{n_{i+2}}, t) = \mathbb{M}(Ty, T^2y, t)$

If $y \neq Ty$ then $\mathbb{M}(Ty, T^2y, t) >\ M(y, Ty, t)$ since T is a FIC map so by definition $\mathbb{M}(y, Ty, t) = \mathbb{M}(Ty, T^2y, t)$But $\mathbb{M}(Ty, T^2y, t) >\ M(y, Ty, t)$ this is contradiction so$y = Ty$.

Since $\mathbb{M}(x_{n+1}, y, t) > M(x_n, y, t) \forall\ n \in N$ so $\{x_n\}$ converges to y then the sequence $\{x_n\}$ converges through a fixed point of T.

Furthermore the sequence $\mathbb{N}(x'_n, x'_{n+1}, t)$ is a sequence which is non-increasing, bounded below by 0 and the subsequence $\{x_n\}$ converging to y'

$\mathbb{N}(y', Ty', t) = \lim_{n\to\infty} \mathbb{N}(x'_{n_i}, x'_{n_{i+1}}, t) = \lim_{n\to\infty} \mathbb{N}(x'_{n_{i+1}}, x'_{n_{i+2}}, t) = \mathbb{N}(Ty', T^2y', t)$

If $y' = Ty'$ then $\mathbb{N}(Ty', T^2y', t) < N(y', Ty', t)$ Since T is a FIC map so by definition $\mathbb{N}(y', Ty', t) = \mathbb{N}(Ty', T^2y', t)$But $\mathbb{N}(Ty', T^2y', t) < N(y', Ty', t)$ which is a again a conflict so $y' = Ty'$

Now because $\mathbb{N}(x_{n+1}, y', t) < N(x_n, y', t)\ \forall n$ so $\{x_n\}$ converges to y then the sequence $\{x_n\}$ converges to a fixed point which is of T.

Theorem 2.4: Let $(X, \mathbb{M}, \mathbb{N}, *, \lozenge)$ is Ifms and $T: X \to X$ is a continuous FIC map and the sequence of iterates $\{x_n\}$ in Ifms defined by $x_{n+1} = Tx_n n \in N$ for $x \in X$ has a subsequence converging to a point that T possess a fixed point.

PROOF: Let the sequence $\{\mathbb{M}(x_{n+1}, x_n, t\}$ is a sequence of real numbers which is non-decreasing, bounded above by 1 so there exist a limit. In view of the subsequence converges to y and T is continuous on X therefore Tx_n converges to Ty and $T^2(x_{n_i})$ converges to T^2y

$\mathbb{M}(y,Ty,t)=\lim_{n\to\infty}\mathbb{M}(x_{n_i},x_{n_{i+1}},t)=\lim_{n\to\infty}\mathbb{M}=\mathbb{M}(Ty,T^2y,t)$
$1\geq\ \mathbb{M}(Ty,T^2y,t)\geq\ \mathbb{M}\left(y,Ty,\frac{t}{k}\right)$
$\geq\ \mathbb{M}\left(y,Ty,\frac{t}{k^2}\right)\geq\cdots$
$\geq\ \mathbb{M}\left(y,Ty,\frac{t}{k^n}\right)\ \forall n\in N\ 0<k<1$
When $n\to\infty\mathbb{M}\left(y,Ty,\frac{t}{k^n}\right)\to 1$ consequently $\mathbb{M}\left(y,Ty,\frac{t}{k}\right)=1$Thus $y=Ty$.
Similarly let the sequence $\{\mathbb{N}(x'_{n+1},x'_n,t\}$ is non-increasing sequence of reals, bounded below by 0 so it possess a limit by reason of the subsequence converges to y' and T is continuous on X therefore Tx'_n converges to Ty and $T^2(x'_{n_i})$ converges to T^2y'.
Thus
$\mathbb{N}(y',Ty',t)=\lim_{n\to\infty}\mathbb{N}(x'_{n_{i+1}},x'_{n_{i+2}},t)=\mathbb{N}(Ty',T^2y',t)$
$0\leq\ \mathbb{N}(Ty',T^2y',t)\leq\mathbb{N}\left(y',Ty',\frac{t}{k}\right)\leq\mathbb{N}\left(y',Ty',\frac{t}{k^2}\right)\leq\ldots$
$\leq\mathbb{N}\left(y',Ty',\frac{t}{\mathrm{k}^n}\right)\ \forall n\in N\quad 0<k<1$
When $n\to\infty$, $\mathbb{N}\left(y',Ty',\frac{t}{k^n}\right)\to 0\Rightarrow\mathbb{N}\left(y',Ty',\frac{t}{k}\right)=0$ thus $y'=Ty'\Rightarrow$ T acquires a fixed point.
NOTE 2.5: If T is a FIC map whichever is not continuous, then it may not have a fixed point and if T is not a fuzzy iterated contraction but it is continuous then T may or may not have a fixed point.
NOTE 2.6: It is sufficient to show that fuzzy iterated contraction should be continuous but not necessary.
NOTE 2.7: In complete Ifms if T is not a contraction but few powers of T is contraction, then T possess a fixed point which is unique.
Theorem 2.8: Let $(X,\mathbb{M},\mathbb{N},*,\lozenge)$ be a Ifms where $*$ is t-norm and $\lozenge$ is co-norm. If $T:X\to X$ be a FIC map if for few powers of T say T^p is a FIC map, i.e.
$\mathbb{M}(T^px,(T^p)^2x,t)\geq\mathbb{M}(x,T^px,\frac{t}{k})$
$\mathbb{N}(T^px,(T^p)^2x,t)\leq\mathbb{N}(x,T^px,\frac{t}{k})$
If T^p is continuous at y where $y=lim\ (T^p)^n$ for any arbitrary $x\in X$ then T possess a fixed point.
PROOF: In view of T is a fuzzy iterated contraction that is continuous at y.
$\mathbb{M}(T^px,(T^p)^2x,t)\geq\mathbb{M}\left(x,T^px,\frac{\mathrm{t}}{k}\right)\quad 0<K<1$
$\mathbb{N}(T^px,(T^p)^2x,t)\leq\mathbb{N}(x,T^px,\frac{t}{k})\quad 0<K<1$
$$\mathbb{M}((T^p)^nx,(T^p)^mx,t)\geq\prod_{i=m}^{n}\mathbb{M}\left((T^p)^{i+1}x,(T^p)^ix,\frac{t}{n-m}\right),\quad\forall n>m,\qquad n,m\in N$$
$$\geq\prod_{i=m}^{n}\mathbb{M}\left(x,T^px,\frac{t}{(n-m)*k^{i-1}}\right)\to 1\ (m\to\infty)$$
$$\mathbb{N}((T^p)^nx,(T^p)^mx,t)\leq\prod_{i=m}^{n}\mathbb{N}\left((T^p)^{i+1}x,(T^p)^ix,\frac{t}{\mathrm{n}-m}\right),\qquad\forall n>m,\qquad n,m\in N$$
$$\leq\prod_{i=m}^{n}\mathbb{N}\left(x,T^px,\frac{t}{(n-m)*k^{i-1}}\right)\to 0\ (m\to\infty)$$
Therefore $\{(T^p)^n\}_{n=1}^{\infty}$ is definitelya Cauchy sequence in X and $(X,\mathbb{M},\mathbb{N},*,\lozenge)$ is complete metric space. Thus $\exists\ y\in X$ such that $y=lim_{n\to\infty}(T^p)^nx$ and (T^p) is continuous
Consequently
$T^p(y)=\lim_{n\to\infty}T^p((T^p)^ny)=\lim_{n\to\infty}(T^p)^{n+1}x=y$
It is quite obvious to show that $d(y,fy)\leq k^pd(y,fy)$ since $k^p\leq 1$
Therefore $d(y,fy)=0$ so by definition of metric space $y=fy$ and f possess a fixed point.
NOTE 2.9: If T is continuous FIC map on complete Ifms has a fixed point if T is not continuous it may have multiple fixed points.
Theorem 2.10: Let $(X,\mathbb{M},\mathbb{N},*,\lozenge)$ is complete Ifms and $T:X\to X$ is a FIC map then sequence of iterates x_n converges to$y\in X$. In that case, T is continuous at y, then y = Tyi.e. T possess a fixed point.

PROOF: Let $x_{n+1} = Tx_n n \in N$It is quite obvious to show that $\{x_n\}$ is Cauchy sequence, after allT is a fuzzy iterated contraction, Cauchy sequence converges to $y \in X$ Since X is Complete Ifms. Moreover if T is continuous at y then$x_{n+1} = Tx_n$. It follows that $y = Ty$.

CONCLUSION

Here in this paper we have deduced a few fixed point results in intuitionistic fuzzy metric space for fuzzy iterated contraction maps. This result can be further used for getting solution of various problems for iterated fuzzy maps defined if image processing, biological problems, information technology etc. This result can be further used for getting fixed points of various fuzzy iterated maps in different spaces.

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