Journal of Contemporary Issues in Business and Government Vol. 27, No. 2, 2021
https://cibg.org.au/

P-ISSN: 2204-1990; E-ISSN: 1323-6903
DOI: 10.47750/cibg.2021.27.02.298

Coupled fixed points for maps under $(\psi-\alpha-\beta)$ −general Contractive conditions in partially ordered soft metric space

RAMAKANT BHARDWAJ[1a,b], QAZI AFTAB KABIR[2]*, RITU SHRIVASTAVA[3], SUDEEP KUMAR PANDEY[4]

[1a]Department of Mathematics, AMITY university Kolkata (WB), India
[1b]Department of Mathematics, APS University Rewa (MP), India
[2]Department of Mathematics, Govt Gandhi Memorial science college Jammu, India
[3]Department of Mathematics, Bahrain Polytechnique Bahrain
[4]Department of Mathematics, Govt. Degree College Amarpur -Dindori, MP, India
*Corresponding Author

Abstract: In This paper, we prove the some new fixed point theorems for$(\psi-\alpha-\beta)-$ contractive mappings in partially ordered soft metric space which generalize the common unique fixed point theorem to the case of $(\psi-\alpha-\beta)$ −contractive mappings in partially ordered soft metric space with closed bounded set. Our results are the extensions of the results of some well-known recent result in the literature. Obtained results are very useful in Business management and stock marketing.

Keywords: Contractive mappings, Coupled fixed point (CFP), Partial Soft metric space (PSMS), partially ordered Soft metric space (POSMS), Business management

INTRODUCTION

In 1994 Mathews, introduced the notion of a partial metric space as a part of the study of denotational semantics of data flow networks. He also showed that the Banach contraction principle can be generalized to the PMS. Not only in mathematics, have POMS had vast applications in computer science, information science etc. Rao et al. [11] introduced the notion of $(\psi-\alpha-\beta)-$ contraction maps with respect to another mapson a partial metric space.In this paper we introduced $(\psi-\alpha-\beta)$ −contractive mappings in partially ordered soft metric space which generalize the common unique fixed point theorem to the case of $(\psi-\alpha-\beta)$ −contractive mappings.

A thought of soft theory as latest mathematical device is discussed in 1999 Molodtsov [10]. Models of soft set are contributed by several researchers (see [7], [8], [9]). In recent times Wadkar et.al ([12]) shared his plan of FPT in SM space. In this paper the results are the expansion of Wadkar and et.al [5]. In this paper we use some definitions and characteristics of soft set from [5]. Bhardwaj [1] have recently demonstrated some new outcomes for withdrawals in CPMS. Wadker et al. [12] likewise demonstrated coupled fixed point hypothesis in somewhat requested PMS. Several papers have been published containing fixed point results for contractive mappings with different CBS in POMS (see[2, 3, 4 6, 7, 8 and 9]).

In this paper we shall prove new coupled fixed point theorems$(\psi-\alpha-\beta)-$ contractive mappings in partially ordered soft metric space with closed bounded set by employing some notions of wadker et al. [12] and Rao et al. [11] as well as a rational type contractive condition.

Definitions and Preliminaries

Throughout this paper we represent coupled fixed point **(CFP)**, complete partial soft metric space **(CPSMS)**, partial metric space **(PMS)**, partially ordered soft metric space **(POSMS)**.

Now, let us recall some basic concepts and facts about **(POMS)** and **(POSMS).**

Definition 2.1. [12]A partially ordered set is a set p and a binary relation $\leq$ denoted by $(X,\leq)$ such that for all $x,y,z\in p$

i. $x\leq x$, (reflexivity)
ii. $x\leq y,\ y\leq z\Longrightarrow x\leq z$ (transitivity)
iii. $x\leq y,\ y\leq x\Longrightarrow x=y$ (anti − symmetry)

Definition 2.2. [12]let (X,p) be a PMS

i. A sequence $\{x_n\}$ in (X,p) is said to convergence to a point $x\in X$ if and only if $p(x,x)=\lim_{n\to\infty}p(x,x_n)$.
ii. A sequence $\{x_n\}$ in (X,p) is said to be Cauchy sequence if $\lim_{n,m\to\infty}p(x_n,x_m)$. exists and is finite.

Definition 2.3. [12]A metric space(X,d) is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.4. [3] Let(X,d) be a metric space. An element $(X,y)\in X\times X$ is said to be a CFP mapping $F:X\times X\to X$ if $F(\tilde{x},y)=\tilde{x}$ and $F(y,\tilde{x})=y$

Definition 2.5. [2]. A contractive mapping $T: X \to CB(X)$ is called a contraction mapping if there exists $k \in (0,1)$ such that $H(T\tilde{x}, Ty) \leq kd(\tilde{x}, y)\ \forall\, \tilde{x}, y \in X$ and $\tilde{x} \in X$is said to be a fixed point of T if $\tilde{x} \in T(X)$

Definition 2.6. ([10])Let Vbe a universe and Abe a set of parameters. Let P(V) denote the power set of V. A pair (F, A) is called a (ss) over V, where Fis a mapping given by $F: A \to P(V)$.

Definition 2.7. ([7]) A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A^*)$ is said to be a (psms) on the (ss)$\tilde{X}$ if

1) $\tilde{d}(P_\Phi{}^u, P_\psi{}^v) \geq \tilde{0}$ for all $P_\Phi{}^u, P_\psi{}^v \in \tilde{X}$,
2) $\tilde{d}(P_\Phi{}^u, P_\psi{}^v) = \tilde{0}$ if and only if $P_\Phi{}^u = P_\psi{}^v$,
3) $\tilde{d}(P_\Phi{}^u, P_\psi{}^v) = \tilde{d}(P_\Phi{}^v, P_\psi{}^u)$ for all $P_\Phi{}^u, P_\psi{}^v \in \tilde{X}$,
4) $\tilde{d}(P_\Phi{}^u, P_\psi{}^v) \leq \tilde{d}(P_\Phi{}^u, P_\psi{}^w) + \tilde{d}(P_\Phi{}^w, P_\psi{}^v)$ for all $P_\Phi{}^u, P_\psi{}^v, P_\psi{}^w \in \tilde{X}$,

The (ps)$\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a (sms) and denoted by $(\tilde{X}, \tilde{d}, \tilde{E})$ or $(\tilde{X}, \tilde{d})$.

Definition 2.8. Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a(psms)and $\tilde{r}$ be a non negative soft real number. Then the soft set $B(P_\Phi{}^u, \tilde{r}) = \{P_\psi{}^v \in SP(\tilde{X}): \tilde{d}(P_\Phi{}^u, P_\psi{}^v) \tilde{<} \tilde{r}\}$ is called soft open ball with center $P_\Phi{}^u$ and of radius $\tilde{r}$.

Example 2.9. Let $\tilde{A} = \mathbb{N}, X = [0,1]$ and let soft partial metric $\tilde{d}$ be defined as follows:

$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = |\alpha - \alpha'| + |\tilde{p} - \tilde{q}| + |u - v|$$
$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = \tilde{d}(\alpha, \alpha') - \left(\tilde{d}(\alpha, \alpha') - \tilde{d}(\tilde{p}, \tilde{q}) - \tilde{d}|u - v|\right)$$
$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = \tilde{d}(\alpha, \alpha') - \tilde{d}(\alpha, \alpha')$$

Where $\mathbb{N}$ is a natural number of set. Therefore $(\tilde{X}, \tilde{d}, \tilde{E})$ be a (psms)

MAIN RESULTS

Theorem 3.1. Let $(\tilde{X}, d)$ be a POSMS and let mappings $T, S: \tilde{X} \to C(\tilde{X})$

Satisfy the following conditions;

i) For each $\tilde{x} \in \tilde{X}, T(\tilde{x}), S(\tilde{x}) \in CB(\tilde{X})$,
ii) $\psi[H(T(\tilde{x}), S(\tilde{y}))] \leq \alpha_1[d(\tilde{x}, \tilde{y})) + d(\tilde{y}, S(\tilde{y}))] + \alpha_2[d(\tilde{y}, S(\tilde{y})) + d(\tilde{x}, T(\tilde{x}))] + \alpha_3[d(\tilde{y}, T(\tilde{x})) + d(\tilde{x}, S(\tilde{y}))]$

where $\alpha_1, \alpha_2, \alpha_3$ are non-negative real numbers and $\alpha_1 + \alpha_2 + \alpha_3 < \frac{1}{2}$. Then there exists $p \in \tilde{X}$ such that $p \in T(\tilde{x}) \cap S(\tilde{x})$.

Proof. Let $\widetilde{x_0} \in X, T(\widetilde{x_0})$ is a non-empty CBS of X. We can choose that $\widetilde{x_1} \in T(\widetilde{x_0})$,for this $\widetilde{x_1}$by the same reason mentioned above $S(\widetilde{x_1})$ is non-empty closed bounded subset of $\tilde{X}$.

Since $\widetilde{x_1} \in T(\widetilde{x_0})$ and $S(\widetilde{x_1})$ are CBS of X, there exist $x_2 \in S(\widetilde{x_1})$ such that

$$d(\widetilde{x_1}, x_2) \leq H(T(\widetilde{x_0}), S(\widetilde{x_1})) + (\psi - \alpha - \beta) - \lambda,$$

where $\lambda = \max\left\{\frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - (\alpha_1 + \alpha_2 + \alpha_3)}, \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - (\alpha_1 + \alpha_2 + \alpha_3)}\right\}$

$$d(\widetilde{x_1}, x_2) \leq H(T(\widetilde{x_0}), S(\widetilde{x_1})) + \lambda$$
$$\leq \alpha_1[d(\widetilde{x_0}, \widetilde{x_1}) + d(\widetilde{x_1}, S(\widetilde{x_1}))] + \alpha_2\left[d(\widetilde{x_1}, S(\widetilde{x_1})) + d(\widetilde{x_0}, T(\widetilde{x_0}))\right]$$
$$+\alpha_3[d(\widetilde{x_1}, T(\widetilde{x_0})) + d(\widetilde{x_0}, S(\widetilde{x_1}))] + (\psi - \alpha - \beta) - \lambda$$
$$\leq \alpha_1[d(\widetilde{x_0}, \widetilde{x_1}) + d(\widetilde{x_1}, x_2)] + \alpha_2[d(\widetilde{x_1}, x_2) + d(\widetilde{x_0}, \widetilde{x_1})]$$
$$+\alpha_3[d(\widetilde{x_1}, \widetilde{x_1}) + d(\widetilde{x_0}, x_2)] + (\psi - \alpha - \beta) - \lambda$$
$$\leq \alpha_1[d(\widetilde{x_0}, \widetilde{x_1}) + d(\widetilde{x_1}, \widetilde{x_2})] + \alpha_2[d(\widetilde{x_1}, x_2) + d(\widetilde{x_0}, \widetilde{x_1})]$$
$$+\alpha_3[d(\widetilde{x_1}, \widetilde{x_1}) + d(\widetilde{x_0}, \widetilde{x_1}) + d(\widetilde{x_1}, \widetilde{x_2})] + (\psi - \alpha - \beta) - \lambda$$
$$d(\widetilde{x_1}, \widetilde{x_2}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - (\alpha_1 + \alpha_2 + \alpha_3)} d(\widetilde{x_0}, \widetilde{x_1}) + (\psi - \alpha - \beta) - \lambda$$
$$d(\widetilde{x_1}, \widetilde{x_2}) \leq \lambda d(\widetilde{x_0}, \widetilde{x_1}) + (\psi - \alpha - \beta) - \lambda$$

Thus for this $\widetilde{x_2}$, $T(\widetilde{x_2})$ is a non-empty CBS of X.

Since $x_2 \in S(\widetilde{x_1})$ and $S(\widetilde{x_1})$ and $T(\widetilde{x_2})$ are CBS of X, there exist $x_3 \in T(\widetilde{x_2})$

Such that

$$d(\widetilde{x_2}, \widetilde{x_3}) \leq H(T(\widetilde{x_2}), S(\widetilde{x_1})) + (\psi - \alpha - \beta) - \lambda^2$$
$$\leq \alpha_1[d(\widetilde{x_2}, \widetilde{x_1}) + d(\widetilde{x_1}, S(\widetilde{x_1}))] + \alpha_2\left[d(\widetilde{x_1}, S(\widetilde{x_1})) + d(\widetilde{x_2}, T(\widetilde{x_2}))\right]$$
$$+\alpha_3[d(\widetilde{x_1}, T(\widetilde{x_2})) + d(\widetilde{x_2}, S(\widetilde{x_1}))] + (\psi - \alpha - \beta) - \lambda^2$$
$$\leq \alpha_1[d(\widetilde{x_2}, \widetilde{x_1}) + d(\widetilde{x_1}, \widetilde{x_2})] + \alpha_2[d(\widetilde{x_1}, \widetilde{x_2}) + d(\widetilde{x_2}, \widetilde{x_3})]$$
$$+\alpha_3[d(\widetilde{x_1}, \widetilde{x_3}) + d(\widetilde{x_2}, \widetilde{x_2})] + (\psi - \alpha - \beta) - \lambda^2$$
$$\leq \alpha_1[d(\widetilde{x_2}, \widetilde{x_1}) + d(\widetilde{x_1}, \widetilde{x_2})] + \alpha_2[d(\widetilde{x_1}, \widetilde{x_2}) + d(\widetilde{x_2}, \widetilde{x_3})]$$
$$+\alpha_3[d(\widetilde{x_1}, \widetilde{x_2}) + d(\widetilde{x_2}, \widetilde{x_3}) + d(\widetilde{x_2}, \widetilde{x_2})] + (\psi - \alpha - \beta) - \lambda^2$$
$$d(\widetilde{x_2}, \widetilde{x_3}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - (\alpha_1 + \alpha_2 + \alpha_3)} d(\widetilde{x_1}, \widetilde{x_2}) + (\psi - \alpha - \beta) - \lambda^2$$
$$\leq \lambda d(\widetilde{x_1}, \widetilde{x_2}) + (\psi - \alpha - \beta) - \lambda^2$$
$$\leq \lambda\{\lambda d(\widetilde{x_0}, \widetilde{x_1}) + \lambda\} + (\psi - \alpha - \beta) - \lambda^2$$
$$d(x_2, x_3) \leq \lambda^2 d(\widetilde{x_0}, \widetilde{x_1}) + (\psi - \alpha - \beta) - 2\lambda^2$$

Similarly this process continue and we get a sequence $\{\widetilde{x_n}\}$ such that $\widetilde{x_{n+1}} \in S(\widetilde{x_n})$ or $\widetilde{x_{n+1}} \in T(\widetilde{x_n})$ and $d(\widetilde{x_{n+1}}, \widetilde{x_n}) \leq \lambda^n d(\widetilde{x_0}, \widetilde{x_1}) + (\psi - \alpha - \beta) - n\lambda^n$.
Suppose $0 \ll u$ be given, choose that, a natural number N_1 such that $\lambda^n d(\widetilde{x_0}, \widetilde{x_1}) + n\lambda^n \ll u \; \forall \; n \geq N_1$

$$\Rightarrow d(\widetilde{x_{n+1}}, \widetilde{x_n}) \ll u.$$

$\therefore \{\widetilde{x_n}\}$ is a Cauchy sequence in (X, d) is a CPMS, $\exists \; p \in X$ such that $\widetilde{x_n} \rightarrow p$. So choose a natural number N_2 such that

$d(\widetilde{x_n}, p) \ll \frac{u(1-(\alpha_1+\alpha_2+\alpha_3))}{2v(1+(\alpha_1+\alpha_2+\alpha_3))}$ and

$d(\widetilde{x_{n-1}}, p) \ll \frac{u(1-(\alpha_1+\alpha_2+\alpha_3))}{2v(\alpha_1+\alpha_2+\alpha_3)} \forall \; n \geq N_2$.

$$d(T(p), p) \leq d(p, \widetilde{x_n}) + d(\widetilde{x_n}, T(p))$$

$$\leq d(p, \widetilde{x_n}) + H(S(\widetilde{x_{n-1}}), T(p))$$

$$\leq d(p, \widetilde{x_n}) + \alpha_1\left[d(\widetilde{x_{n-1}}, p) + d\big(p, T(p)\big)\right] + \alpha_2\left[d(p, T(p)) + d\big(\widetilde{x_{n-1}}, S(\widetilde{x_{n-1}})\big)\right]$$
$$+\alpha_3[d(p, S(\widetilde{x_{n-1}})) + d(\widetilde{x_{n-1}}, T(p))]$$

$$\leq d(p, \widetilde{x_n}) + \alpha_1\left[d(\widetilde{x_{n-1}}, p) + d\big(p, T(p)\big)\right] + \alpha_2[d(p, T(p)) + d(\widetilde{x_{n-1}}, \widetilde{x_n})]$$
$$+\alpha_3[d(p, \widetilde{x_n}) + d(\widetilde{x_{n-1}}, T(p))]$$

$$\leq d(p, \widetilde{x_n}) + \alpha_1\left[d(\widetilde{x_{n-1}}, p) + d\big(p, T(p)\big)\right] + \alpha_2[d(p, T(p)) + d(\widetilde{x_{n-1}}, p) + d(p, \widetilde{x_n})]$$
$$+\alpha_3[d(p, \widetilde{x_n}) + d((\widetilde{x_{n-1}}, p) + (p, T(p))]$$

$$d(T(p), p) \leq \frac{\alpha_1+\alpha_2+\alpha_3}{(1-(\alpha_1+\alpha_2+\alpha_3)} d(\widetilde{x_{n-1}}, p) + \frac{(1+(\alpha_2+\alpha_3)}{(1-(\alpha_1+\alpha_2+\alpha_3)} d(\widetilde{x_n}, p) \forall \; n \geq N_2.$$

$d(T(p), p) \ll \frac{u}{v}$ for all $v \geq 1$, we get $\frac{u}{v} - d(T(p), p) \in P$ and as $n \rightarrow \infty$, we get $\frac{u}{v} \rightarrow 0$ and P is closed $-d(T(p), p) \in P$ but $d(T(p), p) \in P$. Therefore $d(T(p), p) = 0$ and so $p \in T(p)$.
Similarly it can be established that $p \in S(p)$. Hence $p \in T(p) \cap S(p)$.

REFERENCES

1. Bhardwaj,R, (2012). "Coupled fixed point theorem in partially ordered metric spaces", Mathematical Theory and Modeling, Vol.2 No.4.
2. Das, S. and Samanta, S. K. (2013). Soft metric, Ann. Fuzzy Math. Inform. **6 (1);** 77–94.
3. Kabir, Q.A., Siddhartha, M., Bhardwaj, R., and Mishra, P .K. (2020), New coupled fixed point results in partially ordered metric space for contractive mappings, International journal of Advanced science and Technology, 29(8); 755-760.
4. Kabir, Q. A. Bhardwaj, R., Mohammad, M., Verma, R. &Barve, S.K. An intersection property gluing on weak outwardly hyperconvex spaces. Materials Today: Proceedings 29(2020) 605-610.
5. Kabir, Q. A., Mohammad, M., Jamal, R. and Bhardwaj, R. (2017)'Unique fixed points and mappings in hyperconvex metric spaces', International Journal of Mathematics and its Applications, Vol.5, No. 3-B, pp. 171-177.
6. Matthews, S.G. (1994), Partial metric topology, proc. 8th summer conference on general topology and applications, in: Ann. New York Acad. Sci., Vol. 728, 183-197.
7. Mohammad, M., Jamal, R., Mishra, J., Kabir, Q.A., and Bhardwaj, R.(2018), "Coupled fixed point theorem and dislocated quasi-metric space, International journal of pure and applied mathematics", 10(119), 1249-1260.
8. Mohammad, M. Bhardwaj, R. Kabir, Q. A., Barve, S. K., and Dassani, M. (2020).Fixed point theorems, multi-valued contractive mappings and multi-valued caristi type mappings. Materials Today: Proceedings 29, pp 625-632.
9. Mohammad, M., Jamal, R. and Kabir, Q. A. (2017). Soft cone metric spaces and common fixed point theorems, International Journal of mathematical Archive,**8(9);** 1-6.
10. Molodtsov, D. (1999). Soft set theory-first results, Comput. Math. Appl. **37(4);** 19-31
11. Rao,KPR, Kishore GNV, (2011). A unique common fixed point theorems for four maps under contractive mappings in partially ordered soft metric space which generalize the common unique fixed point theorem to the case of $(\psi - \varphi)$ −contractive conditions in partial metric spaces, Bull. Math. Anal. Appl; 3(3); 56-63.
12. Wadkar,B. R. Sing,B.K. Bhardwaj, R. "Coupled fixed point theorem in partially ordered metric spaces", Mathematical Theory and Modeling, Vol. 3 No.6, 2013.