Journal of Contemporary Issues in Business and Government Vol. 27, No. 2, 2021
https://cibg.org.au/
P-ISSN: 2204-1990; E-ISSN: 1323-6903
DOI: 10.47750/cibg.2021.27.02.207

Evaluating Endogenous Growth

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Abstract: There are three main problems in analyzing the origins of economic growth: First, the choice of irrelevant production functions, second, the failed ignorance of such indispensable variables as human capital in a research model, and third, the use of unreliable econometric methods. Using the Bayesian non-linear regression via the Metropolis-Hasting algorithm, this work estimates a human capital-extended variable elasticity of substitution (VES) function in a Solow-Swan one-sector growth model for Vietnam, an emerging economy. The study reports the following findings: (1) A variable elasticity of substitution between capital and labor (ES) of higher than one implies that the possibility of unbounded endogenous growth has been generated in the economy of Vietnam; (2) Despite a continuous increase in physical investment over the transition period, the labor share rises relative to the capital share in Vietnam due to including embodied human capital in the model; (3) Scale economies do not take place in Vietnam, i.e., the economy approaches a status of perfect competition market; and (4) The human capital-extended VES production model is found to be appropriate for evaluating endogenous growth.

Keywords: Bayesian non-linear regression, Elasticity of substitution between capital and labor (ES), Variable Elasticity of Substitution (VES), Quality evaluating endogenous growth.

INTRODUCTION

Despite many advances attained in growth theory, discussions about the origins of economic growth, specifically the relationship between the ES and economic growth, have seemingly never exhausted. To perform a comprehensive analysis of this link, a relevant function should be specified. Since the appearance in the late 1930s, the Cobb-Douglas production function has experienced a long life without any substitute. However, later, its applicability has been questioned due to the restricted assumptions of the unitary ES and returns to scale, which hides the major role of capital-labor interaction for growth process. Researchers in growth area had sought a more relevant function before the CES showed up (Arrow et al., 1961). The CES specification is less restrictive but the ES is constant, though differs from unity. The CES is admitted to be far from appropriate in accessing a fast-developing economy. The VES production function, where the ES is allowed to vary alongside the process of economic development is more favorable. It is, nevertheless, noted that in the VES functions specified in many works, human capital as a significant determinant is dropped out of analysis (for instance, Antras, 2004; Chirinko & Mallick, 2017; Paul Saumik, 2019; Thach, 2020a).

Like China, the Vietnamese economy has been gaining an impressive expansion since the start of economic reforms (Нгуен, 2003). Capital stock has grown five times faster than labor force in this country (Figure 1). This tendency hypothesizes the rise of the ES in response to an increase in capital-labor ratio. Hence, the use of a VES framework for analyzing Vietnam's economic growth is suggested as reasonable. In the meanwhile, many works in economic transition theory (for instance, Sinelnikov-Murylev et al., 2015; Pham & Ly, 2016) applied the Cobb-Douglas forms, which cannot evidently provide new insights into the specific area of growth theory, where a varying ES is closely connected with economic development. Also, many previous growth studies (for instance, among others, Antras, 2004; Miller, 2008; Chirinko & Mallick, 2017; Muck, 2017; Paul Saumik, 2019) employed the out-of-date frequentist approach that may lead to unreliable outcomes (Kreinovich et al., 2019; Thach et al., 2020). This more traditional approach suffers from shortcomings, in particular, its non-ability to provide direct probabilistic interpretations of estimates. In contrast, the Bayesian framework permits probability statements. More importantly, Bayesian results are the full posterior distributions of particular effects, which help reduce the uncertainty of the model.

For the above reasons, the focus of the current study is to specify a human capital-extended VES production model based on a data set covering 30 years of Vietnam, from which useful implications can be drawn for the growth policy of other emerging economies.

The rest of the article is organized as follows. The analytical framework of the VES production function and empirical research on the VES are presented in Section 2. In Section 3, a human capital-extended VES production model for the economy of Vietnam is estimated using the Bayesian non-linear regression. Section 4 discusses the key findings. The final conclusion and main limitation are provided in Section 5.

CONCEPTUAL BACKGROUND AND EMPIRICS

Conceptual Background

In growth literature, the correspondence among technological advancement, the ES, factor shares and economic growth was first theoretically investigated in a well-known work of Hicks (1932), where in the context of unchanged technology, since capital increases faster than labor, their relative prices reduce and a tendency of substitution of capital for labor occurs. Capital-labor ratio grows until the ES falls below unity. Then, the marginal returns to capital as well as capital share are going to decrease. The economic system approaches its steady state. In order to offset this tendency, as emphasized by Fellner (1954) and Bruton (1956), the speed of technological progress must be high enough to keep the marginal returns to capital from setting in. Labor-saving technical change should take place so that the entire stock of capital is exploited, owing to which productivity of capital will rise. The rise of labor productivity can result from the accumulation of physical capital or human capital. Hence, there is a need to specify a relevant production function to separate the contribution of technology, physical capital and human capital to aggregate output, as well as to explore the role of the ES responding to an increase in capital-labor ratio for growth process.

A large variety of production functions have been introduced over time. The fixed coefficient function or the Harrod-Domar model has some properties such as production factors in a fixed proportion, the zero ES, and constant returns to scale. The Cobb-Douglas specification has popularly employed since its appearance in 1928 as it is easy to fit the logarithmic function through a normal regression. However, with a too restrictive proposition, that is, the unitary ES, this functional form has attracted a lot of criticism. Many studies discovered that the ES might be different from unity (for instance, Pereira, 2003; Klump et al., 2007; Leon-Ledesma et al., 2010; Mallick, 2012; Muck et al., 2015; Karabarbounis & Neiman, 2014; Thach, 2020c). A less restrictive and more fruitful standard CES production function with the ES value ranging from zero to infinite was introduced by Arrow et al. (1961). The CES, characterized as linear and homogeneous, has constant returns to scale. This functional form, nevertheless, has restrictions too. Similar to the earlier functions such as fixed coefficient and Cobb-Douglas, the CES is attributed a constant ES. Growth researchers were seeking a more generalized functional form, where the ES might vary along with economic progress. A standard VES framework is first proposed by Revankar (1971a). Contrast to the CES or Cobb-Douglas, in a specific VES function developed in the present work, where simple labor is replaced by human capital-augmented labor, we examine the impact of variations in capital-labor ratio on the ES. These variations feed back in the economy, influencing the processes of capital accumulation and economic growth. In this context, the VES production model exhibits the possibility of unbounded endogenous growth even in presence of nonreproducible factor inputs and absence of exogenous technical change (Jones & Manuelli, 1990, 1997; Karagiannis et al., 2005).

Based on Sato & Hoffman (1961), Revankar (1971a), Karagiannis et al. (2005), and Thach (2020a), let us specify the VES production function as follows:

$$Y = TK^{\alpha\varepsilon}[L + \alpha\beta K]^{(1-\alpha)\varepsilon}, \quad (1)$$

where $Y, K,$ and L denote output, capital input, and labor input, respectively, ε denotes returns to scale, T is technology, α, β are parameters.

With an assumption of $\varepsilon = 1$. Eq. (1) is transformed to an intensive form:

$$y = Tk^{\alpha}[1 + \alpha\beta k]^{(1-\alpha)}, \quad (2)$$

where $y = f(k), y \equiv \frac{Y}{L}, k \equiv \frac{K}{L}$.

Eq. (1) is differentiated as follows:

$$f'(k) = \alpha\frac{y}{k} + \alpha(1-\alpha)\beta\frac{y}{1+\alpha\beta k} \quad (3)$$

The second-order differentiation of Eq. (2) results in:

$$f''(k) = T\alpha(1-\alpha)(1+\alpha\beta k)^{-\alpha-1}k^{-1}. \quad (4)$$

$f(k)$ obtains the features of a neoclassical production function in case $f(k) > 0, f'(k) > 0$, and $f''(k) < 0 \; \forall k > 0$, as long as $\beta > -1, T > 0, 0 < \alpha \leq 1$, and $k^{-1} \geq -\beta$.

In case $\beta = 0$, Eq. (2) reduces to the Cobb-Douglas function, but to an AK type if $\alpha = 1$,

The limiting properties of Eq. (2) are:

$$\lim_{k\to 0} f(k) = 0, \lim_{k\to\infty} f(k) = \infty \text{ if } \beta > 0,$$
$$\lim_{k\to -\beta^{-1}} f(k) = E(-\beta)^{-\alpha}(1-\alpha)^{1-\alpha} > 0 \text{ if } \beta < 0. \quad (5)$$

Eq. (3) is specialized to:

$$\lim_{k\to 0} f'(k) = \infty, \lim_{k\to\infty} f'(k) = T(\alpha\beta)^{1-\alpha} > 0 \text{ if } \beta > 0,$$
$$\lim_{k\to -\beta^{-1}} f'(k) = T[-\beta(1-\alpha)^{1-\alpha} > 0 \text{ if } \beta < 0. \quad (6)$$

Hence, in case $\beta > 0$, one of the Inada conditions will not hold. This violation implies that the marginal returns to capital is strictly bounded from below. That is, if $\beta > 0$, then we obtain:

$$\lim_{L\to 0} F(K,L) = (\alpha\beta)^{1-\alpha} > 0. \quad (7)$$

Relying on Eq. (2), the approximation of the labor share gives:

$$s_L = \frac{1-\alpha}{1+\alpha\beta k}, \text{ where } \lim_{k\to 0} s_L = 1-\alpha,$$
$$\lim_{k\to\infty} s_L = 0 \text{ if } \beta > 0 \text{ and } \lim_{k\to -\beta^{-1}} s_L = 1 \text{ if } \beta < 0. \quad (8)$$

The capital share is:

$$s_K = 1 - s_L = \frac{\alpha + \alpha\beta k}{1+\alpha\beta k}. \quad (9)$$

Thus, the ES in Eq. (2) is estimated as follows:

$$\sigma(k) = 1 + \beta k. \quad (10)$$

In sum, $\sigma < 1$ in case $\beta < 0$ and $\sigma > 1$ in case $\beta > 0$. That is, the ES varies with economic development proxied by capital-labor ratio. As documented in a great amount of conceptual and empirical works, the ES in turn impacts on the process of economic development. Jones & Manuelli (1990, 1997) argue that endogenous growth is realistic as long as the marginal returns to capital are strictly bounded from below even without exogenous technical change in presence of a nonreproducible factor. Palivos & Karagiannis (2004) affirm that the ES becoming asymptotically higher than one (along with growing k) is necessary and sufficient for the emergence of a lower bound on the marginal returns to capital. Therefore, in case $\sigma > 1$, along with the rise of k, the production model exhibits the possibility of unbounded endogenous growth.

Empirical Research on VES

Since coming into being, due to a flexible ES, the VES has taken advantage over the Cobb-Douglas and CES and has come into widespread use in growth field. In general, the empirical models applying the VES framework are classified into two main types: using time-series data and using cross-section data.

Time-series data are modeled by several growth economists. Accessing the private nonfarm sectors of the U.S. and Japan, Sato & Hoffman (1968) reveal that the VES is preferred to the CES. Analyzing data on the Japanese private nonfarm, Revankar (1971b) negated the Cobb-Douglas specification in favor of the VES. Unlike two above studies, both the VES and the CES are advocated for the U.S. manufacturing sector by Lovell (1973a). In a study of Roskamp (1977), the ES is estimated for the VES and the CES relying on a data on 38 manufacturing industries of Germany. Nonetheless, a later work on 16 two-digit U.S. manufacturing industries, Lovell (1968) favors the VES form rejecting the Cobb-Douglas and CES. Bairam (1989, 1990) elaborating upon the Soviet and Japanese economies supports the VES function in the rejection of the Cobb-Douglas. In a recent study of Khan & Shrafat (2015), the variable ES is analyzed as a primary determinant of bank growth in Pakistan. Applying an econometric analysis to yearly data on 20 Japanese industrial sectors for the 1970–2012 period, Paul Saumik (2019) chooses a VES specification. Proceeding further in this respect, similar to Brianzoni et al. (2012), Grassetti & Hunanyan (2018) examine the impact of the ES on economic growth within the VES framework, whereas Michetti (2013) considers a sigmoidal production function for the Solow–Swan model. As concluded by these authors, fluctuations and complex dynamics might appear in the case of a sufficiently low ES. Thach (2020b) specifies an aggregate unrestricted VES function to evaluate endogenous growth in the Vietnamese nonfinancial sector on a corporate data set for 2008–2018. Note that in most of the above considered works, except for Bairam (1989), Roskamp (1977), the ES is found to be lower than unity.

Besides time-series models, many of researchers employ cross-section data. Based on the time series data of 17 U.S. two-digit manufacturing industries, Lu & Fletcher (1968) reject the CES in favor of the VES production function in 7 to 9 industries. Also resting on a dataset of the U.S. manufacturing industries, Revankar (1971b) decides in favor of the VES for 5 of 12 industries. Considering the Cobb-Douglas and the CES misspecified, Lovell (1973b) is supportive of the VES using data on 3 out of 17 U.S. two-digit manufacturing industries. The VES framework is revealed to be more appropriate than the CES in most cases (Kazi, 1980). Utilizing corporate data on U.S. two-digit manufacturing firms, in a study by Diwan (1970), the Cobb-Douglas and CES forms are rejected in favor of the VES. An analogous outcome based on agricultural experimental data is obtained by Meyer & Kadiyala (1974). Without a purpose of model comparison, Tsang & Yeung (1976), Zellner & Ryu (1998) estimate the VES and CES functions for the food and kindred products and transportation equipment industries of the U.S., where the ES is greater than one. On a data sample of 82 economies, the ES is found to be greater than one in Karagiannis et al. (2005). The above findings are not in accord with Lu & Fletcher (1968) and Kazi (1980), in which the ES is found to be lower than one.

Notably, the majority of previous investigations on the VES production function made use of the out-of-date frequentist methodology. Furthermore, the results are contradictory with some studies revealing the below-unity ES and others finding the ES to be above unity. More importantly, most of the mentioned studies do not incorporate human capital in the research model and hence cannot explore the critical role of this significant element for endogenous growth.

METHODOLOGY

Bayesian Simulation Algorithm and Specification of Model

For the last three decades, the Bayesian framework has applied more and more commonly in behavioural and social research, while classical econometrics is facing an increasing crisis (Escobar et al., 2018; Nguyen et al., 2018; Tuan et al., 2019; Nguyen et al., 2019; Anh et al., 2018; Sriboonchitta et al., 2019; Thach et al., 2019; Svitek et al., 2019; Kreinovich et al., 2019; Thach et al., 2020; Thach, 2020a; Thach, 2020b; Thach, 2020c). Firstly, unlike frequentist statistical inference, the Bayesian approach is superior in that Bayesian results are not point estimates, but an entire probability distribution of a particular parameter. Thus, the issue of statistically non-significant, but potentially important, effects not being reported is solved in Bayesian analysis (Cohen, 1994; Starbuck, 2006). Secondly, the Bayes rule is applied to all parametric models, whereas a frequentist method designed for a model class cannot as usual be used for other classes. Thirdly, in Bayesian analysis posterior distributions are more balanced and more reliable owing to updating available data with prior knowledge. Particularly, weakly informative priors in Bayesian inference allows for regularizing parameter estimates, while flat priors may trigger type I and type M errors (Lemoine, 2019). This middle view can minimize the effect of a small sample size, making results more persistent in Bayesian analysis.

The current study uses the Bayesian nonlinear regression to estimate a human capital-extended VES production function for Vietnam. In growth literature, the Bayesian nonlinear models are appropriate for the accessment of economic growth (Davidian & Giltinan, 1995; Pinheiro & Bates, 2000). In order to analyze the properties of Vietnamese economic growth, we suggest a VES framework rather than other specifications. A reason for this is that the economy of Vietnam has experienced relatively fast growth over the transition period. Capital-labor ratio has increased in a quickly developing country. According to official statistical sources (Vietnam General Statistics Office, 2018, 2019; Federal Reserve Bank of St. Louis, 2019), during the 30-year market-oriented reform, on average, the growth rate of gross domestic product, capital and labor of Vietnam obtains 6.9, 9.7, and 1.9 percent, respectively, while that of HDI is 1.3 percent (Figure 1). On average, stock of capital rises five times faster than labor force. The higher rate of capital accumulation as compared to labor is assumed to have influenced capital-labor interaction, which in turn has induced increases in the ES.

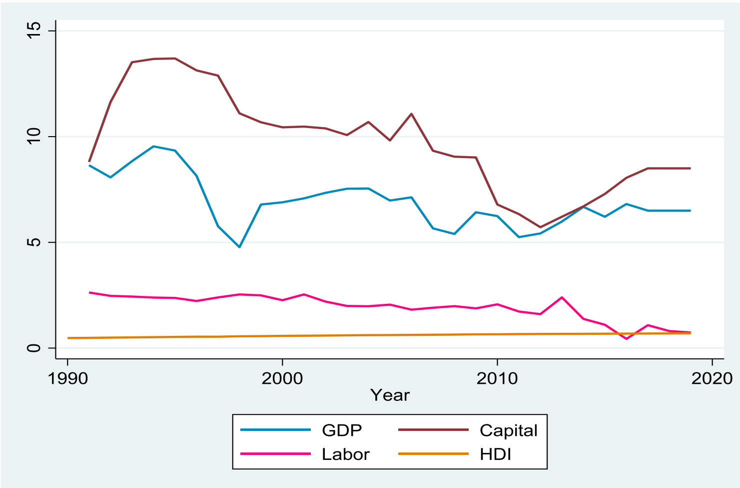

Figure 1: HDI and growth rates of key aggregates. Source: The author's calculations

The VES is superior to others such as the CES or Cobb-Douglas not only because the VES is more relevant to consideration of a dynamic economy but also allows for analysis of scale economies. Therefore, in this article we specify two aggregate VES production functions, unrestricted (nonconstant returns to scale) and restricted (constant returns to scale). Relying on Eq. (1), we conducted two Bayesian simulations utilizing the same macro data on gross domestic product, capital, and labor of Vietnam, in which due to the skewed distributions of the data, the non-linear function is transformed to a logarithmic form.

Prior selection is an important step in Bayesian inference. As spelled out above, we account for weakly informative priors as they exert regularizing effects on posterior probability distributions. So, we set weakly informative priors on the parameters of interest $\alpha, \varepsilon, \beta$. Contrary to Thach (2020b), as parameter β may have a non-positive sign, a normal (0,1) prior will be reasonable for it. Regarding the rest of less important parameters, we pick vague priors. Our Bayesian model is fitted as follows:

Likelihood:

$$lngdp = b0 + \alpha\varepsilon lnK + (1-\alpha)\varepsilon \ln(L + \alpha\beta K), \quad (11)$$

Priors:

$$\begin{gathered} b0 \sim N(0,100) \\ \varepsilon \sim N(0,1) \\ \alpha \sim uniform(0,1) \qquad (12) \\ \beta \sim N(0,1) \\ sig2\_0 \sim Igamma(0.001, 0.001), \end{gathered}$$

where $lngdp, lnK, K,$ and L are natural log of output, natural log of capital, capital, and labor, respectively.

Data Description

The research utilizes yearly time series for the 1990-2019 period on gross domestic product, capital, and labor of Vietnam to specify a human capital-extended VES production function with nonconstant returns to scale (unrestricted VES) and constant returns to scale (restricted VES). Data on gross domestic product and capital at constant 2011 prices are collected from the Penn World Tables (Federal Reserve Bank of St. Louis, 2019). The database of the General Statistics Office of Vietnam (2008; 2019) provides the number of members in the labor force. Units of gross domestic product and capital are billions of U.S. dollars and labor – millions of workers. The label *Ht* is attached to the stock of human capital in year *t*, where *Ht* represents the HDI in year *t*. Furthermore, we define human capital adjusted labor supply as *LHDIt* = *Ht* × *Lt*. In the estimation of the VES function, we will employ *LHDI* as a measure of labor input. The HDI of Vietnam is extracted from the database of the United Nations Development Programme (2020). It is a composite index consisting of three components: longevity, education, and income per capita and thus outlining a more general outlook of human development.

EMPIRICAL RESULTS

Model Comparison

In this subsection, we compare two VES production models, one with nonconstant returns to scale and the other with constant returns to scale. For this, we perform two tests: Bayesian information criteria (Table 1) and Bayes model test (Table 2). As a result, the restricted VES model has smaller DIC, but higher log(ML), log(BF), and P(My) than the unrestricted VES model, and thus is preferrable over the the latter. We will choose the restricted

$(\varepsilon = 1)$ VES model for further analysis.

Table 1: Bayesian information criteria

Model	DIC	log(ML)	log(BF)
Unrestricted VES	-1173	51.68361	.
Restricted VES	-1218.851	54.38875	2.705144

Source: The author's computation.

Table 2: Bayes model test

	log(ML)	P(M)	P(My)
Unrestricted VES	51.6836	0.5000	0.0627
Restricted VES	54.3888	0.5000	0.9373

Source: The author's computation.

Bayesian Posterior Results

The simulation outcome of the restricted VES production model will be examined in this subsection. The model summaries in Table 3 report the following. First, posterior median and mean as key point estimates are almost identical, showing symmetric posterior distributions. Second, the credible intervals for all the model parameters $\alpha, b0, \beta$ and sig2_0 do not contain zero and hence, it can be claimed that variables capital, labor, technology, and beta have strongly positive effects on the model outcome. And finally, for all the model parameters, the standard deviation estimates are small, while Monte-Carlo standard errors (MCSEs) are close to one decimal. Both point estimators indicate the sufficiently high preciseness of the parameter mean estimates.

Table 3: Bayesian posterior simulations

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]
alpha					
b0	1.450663	0.0634865	0.008829	1.461073	[1.284542, 1.539077]
beta	0.7782052	0.5027293	0.089258	0.6233341	[0.1669994, 2.070916]
sig2_0	0.0005857	0.0001665	2.4e-06	0.0005609	[.0003402, 0.0009999]

Source: The author's computation.

In the application of MCMC algorithms, MCMC chain must converge to a stationary distribution so that Bayesian posterior estimates are robust. The rate of acceptance and average efficiency, which impact on MCMC convergence, are 0.58 and 0.23, respectively. These initial indicators are higher than the warning levels of 0.1 and 0.01, respectively. Furthermore, it is essential to inspect the convergence of the MCMC sequences. In case we have the MCMC convergence established, the model parameters will have converged to reasonable persistent values. We perform a test via cusum plots: the more jagged cusum lines, the better MCMC chain mixes. Regardless of how long the MCMC iterations may take, once the mixing of the MCMC chain is sufficiently good, the joint posterior distribution will be sufficient to extract marginalized distributions through the integration of nuisance parameters. Figure 2 demonstrates the jagged cusum lines related to the parameters (alpha, b0, beta, and sig2_0). We can conclude that the test indicates non sign of non-convergence.

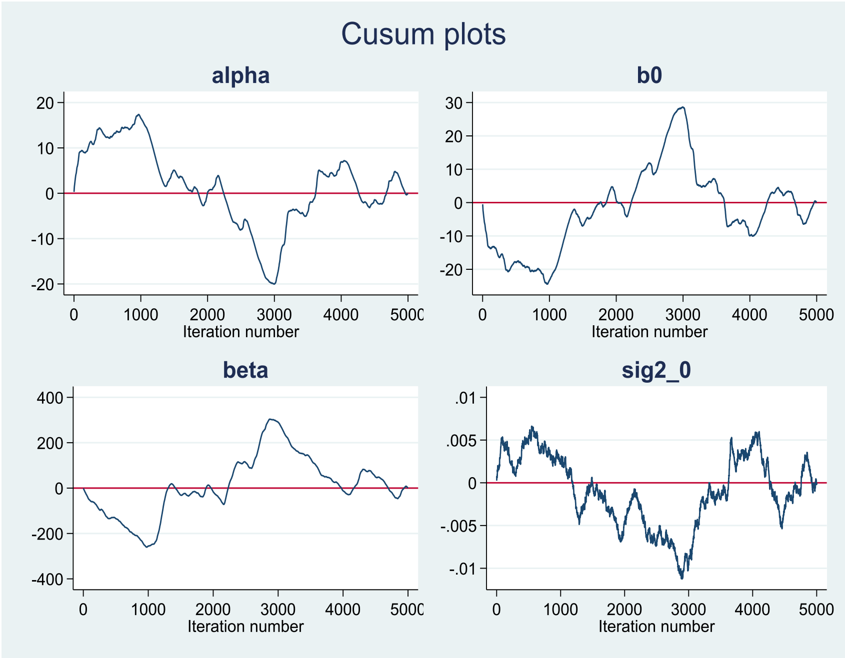

Figure 2: Cusum plots of model parameters. Source: The author's computation

Discussion of Findings

The Bayesian posterior results recorded in Table 3 indicate that all the mean estimates are positive and different from zero. Moreover, they obtain economically plausible meanings. Importantly, as a result of sensitive analysis, the restricted VES model, i.e. the model with constant returns to scale is selected. That implies that effect of scale economies does not exist in the economy of Vietnam. Except for (Karagiannis et al., 2005), no research where unrestricted and restricted VES functions are compared has been found. Our work is the first to compare the two human capital-extended VES production models for a transformation economic system.

More interestingly, the most crucial finding corresponding to our tested hypothesis is that the mean estimate of parameter β (beta) obtains a positive sign. Its estimate of 0.67 seems minor but certainly has a potentially large effect on the ES value, accounting for Eq. (10): $\sigma(k) = 1 + \beta k$. As such, the empirical result shows that the ES estimate is greater than unity for the economy of Vietnam. Our finding is consistent with the predictions of endogenous growth theory (Romer, 1986, 1987), where human capital enormously contributes to sustainable economic growth, with many empirical considerations as well (for instance, Roskamp, 1977; Bairam, 1989; Grassetti et al., 2018; Thach, 2020a). Under the condition of the above-unity ES, while k is growing, the model exhibits the possibility of endogenous growth. This finding means that Vietnam has created the possibility of endogenous growth over the transformation period. Human capital, technical change embodied in physical investment, enhancement of corporate R&D, deepening international integration, and learning-by-doing effect evidently contribute to endogenous economic growth in Vietnam.

As shown in (8) and (9), the factor shares are dependent on the values of $K, L, \alpha,$ and β. Since the β estimate is positive and different from zero, the factor shares change with capital-labor ratio. However, contrast to Thach (2020a), in our current model, the labor share tends to increase significantly over years. This is because the labor share is composed of the combined income of simple labor and embodied human capital.

Finally, for dynamically growing economies like Vietnam, the human capital-extended VES model is shown to be relevant to evaluation of endogenous growth.

CONCLUSIONS

This study specifies a human capital-extended VES production function in a Solow-Swan one-sector growth model for the economy of Vietnam, which have undergone a 30-year market-oriented transition. As compared to frequentist analysis, the Bayesian nonlinear regression used to specify the extended VES function in this work produces the entire posterior distribution of the parameters, and thus reduces model uncertainty and increases its validity. The empirical results demonstrate that the estimate of the variable ES is more than one, suggesting the possibility of endogenous growth generated in the Vietnamese economy over the transformation period, where the growth rate of capital stock has so far outperformed that of labor force. In real life, the rise of the crucial determinants affecting endogenous growth such as accumulation of physical and human capital, technology transfers, and institutional quality is obviously observed in Vietnam. One more important finding of our research is that the human capital-extended VES framework is likely be an appropriate tool to evaluate production technology in such rapidly-developing economies as Vietnam. These findings are accordant with a satisfactory description of data employed in the study and concepts of endogenous growth.

In light of the obtained empirical results, the study suggests some policy implications. In order to acquire the real possibility of endogenous growth, Vietnam must carry out a policy of forced investment in combination with fostering technological advancement. For these combined goals, the country should incentivize formation of physical and human capital, enhance R&D, and in particular, focus on hi-tech transfers. In spite of a fast increase in investment during three last decades, the share of the private sector in the aggregate capital is small in Vietnam. Hence, private firms should more encourage transfers of advanced technology, reinforce production capabilities, and conduct R&D on their own, while the scale of public capital must be minimized. A reasonable policy of attracting foreign investment will help disseminate positive spillover effect to domestic firms. Most importantly, human capital needs to be extensively accumulated in many ways so that the overwhelming majority of population can access educational and medical services. The above mentioned measures are helpful for Vietnam as well as other developing countries.

The main limitation of the present study may be that only human capital embodied in labor is included in our model, whereas incorporating aggregate human capital is necessary. Therefore, future research is motivated to include both aggregate and embodied human capital in a production function.

REFERENCES

1. Anh, Ly H., Vladik Kreinovich, and Nguyen Ngoc Thach, eds. 2018. Econometrics for Financial Applications. Cham: Springer.
2. Antras, P. 2004. Is the U.S. Aggregate Production Function Cobb–Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics*, 4.
3. Arrow, Kenneth J., Hollis B. Chenery, Bagicha Singh Minhas, and Robert M. Solow. 1961. Capital Labour Substitution and Economic Efficiency. *The Review of Economics and Statisticss*, 63: 225–50.
4. Bairam, Erkin. 1989. Learning-by-doing, variable elasticity of substitution and economic growth in Japan, 1878–1939. *Journal of Development Studies* 25: 344–53.
5. Bairam, Erkin. 1990. Capital-labour substitution and slowdown in Soviet economic growth: A re-examination. *Bulletin of Economic Research* 42: 63–72.
6. Brianzoni, Serena, Cristiana Mammana, and Elisabetta Michetti. 2012. Variable elasticity of substituition in a discrete time solow–swan growth model with differential saving. *Chaos, Solitons & Fractals* 45: 98–108. https://doi.org/10.1016/j.chaos.2011.10.004
7. Bruton, Henry J. 1956. Innovations and equilibrium growth. *Economic Journal* 66: 455–66.
8. Chirinko, R. S. and D. Mallick. 2017. The Substitution Elasticity, Factor Shares, and the Low-Frequency Panel Model. *American Economic Journal: Macroeconomics*, 9(4): 225–253. https://doi.org/10.1257/mac.20140302
9. Cohen, Jacob. 1994. The earth is round (p <.05). *American Psychologist* 49: 997–1003.
10. Davidian, Marie, and David M. Giltinan. 1995. *Nonlinear Models for Repeated Measurement Data*. Boca Raton: Chapman & Hall/CRC.
11. Diwan, Romesh K. 1970. About the growth path of firms. *American Economic Review* 60: 30–43.

12. Escobar M.O., Kreinovich V., Nguyen T.N.: Is It Legitimate Statistics or Is It Sexism: Why Discrimination Is Not Rational. In: Anh L., Dong L., Kreinovich V., Thach N. (eds) Econometrics for Financial Applications. ECONVN 2018. *Studies in Computational Intelligence*, vol 760. Springer, Cham. https://doi.org/10.1007/978-3-319-73150-6_18 (2018)
13. Federal Reserve Bank of St. Louis. 2019. Penn World Table. https://fred.stlouisfed.org
14. Fellner, William. 1954. Full use of underutilization: appraisal of long-run factors other than defense. *American Economic Review* 44: 423–26.
15. General Statistics Office of Vietnam. 2008. *Statistics Yearbook of Vietnam 2018*. Hanoi: Statistical publishing house.
16. General Statistics Office of Vietnam. 2019. *Statistical Yearbook of Vietnam 2019*. Hanoi: Statistical publishing house.
17. Grassetti, Francesca, and Gevorg Hunanyan. 2018. On the economic growth theory with kadiyala production function. *Communications in Nonlinear Science and Numerical Simulation* 58: 220–32.
18. Grassetti, Francesca, Cristiana Mammana, and Elisabetta Michetti. 2018. Substitutability between production factors and growth. An analysis using VES production functions'. *Chaos, Solitons & Fractals*, 113: 53–62.
19. Hicks, John. 1932. *The Theory of Wages*. London: Macmillan and Co., Ltd.
20. Huy, D.T.N. (2012). Estimating Beta of Viet Nam listed construction companies groups during the crisis, Journal of Integration and Development, 15(1).
21. Huy, D.T.N. (2011). Estimating Beta of Viet Nam Listed Public Utilities, NAtiral Gas and Oil, Economic and Business Review, 15(1).
22. Jones, Larry E., and Rodolfo Manuelli. 1990. A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy* 98: 1008–38.
23. Jones, Larry E., and Rodolfo E. Manuelli. 1997. Sources of growth. *Journal of Economic Dynamics and Control* 21: 75–114.
24. Karabarbounis, L. and B. Neiman. 2014. "The Global Decline of the Labor Share," The Quarterly Journal of Economics, 129, 61–103.
25. Karagiannis, Giannis, Theodore Palivos, and Chris Papageorgiou. 2005. Variable Elasticity of Substitution and Economic Growth: Theory and Evidence. *In New Trends in Macroeconomics*. Edited by Diebolt Claude. and C. Kyrtsou Catherine. Berlin/Heidelberg: Springer, pp. 21–37.
26. Kazi, Umar A. 1980. The variable elasticity of substitution production function: A case study from Indian manufacturing industries. *Oxford Economic Papers* 32: 163–75.
27. Khan, Anam, Mehmood, Bilal, and Shrafat, Ali Sair. 2015. The variable elasticity of substitution production function: A case study for Pakistani banking sector. *Science International (Lahore)* 27: 6349–52.
28. Klump, R., P. McAdam, and A. Willman. 2007. Factor Substitution and Factor Augmenting Technical Progress in the US: A Normalized Supply-Side System Approach, *Review of Economics and Statistics*, 89, 183–192.
29. Kreinovich Vladik, Nguyen Ngoc Thach, Nguyen Duc Trung, and Dang Van Thanh, eds. 2019. *Beyond Traditional Probabilistic Methods in Economics*. Cham: Springer.
30. Lemoine, Nathan P. 2019. Moving beyond noninformative priors: why and how to choose weakly informative priors in Bayesian analyses. *Oikos* 128: 912–28.
31. Leon-Ledesma, M. A., P. McAdam, and A. Willman. 2010. Identifying the Elasticity of Substitution with Biased Technical Change. *American Economic Review*, 100, 1330–1357.
32. Lovell, CA Knox. 1973a. CES and VES production functions in a cross-section context. *Journal of Political Economy* 81: 705–20.
33. Lovell, CA Knox. 1973b. Estimation and prediction with CES and VES production functions. *International Economic Review* 14: 676–92.
34. Lu, Y., and L.B Fletcher. 1968. A generalization of the CES production function. *Review of Economics and Statistics* 50: 449–52.
35. Mallick, D. 2012. The Role of the Elasticity of Substitution in Economic Growth: A Cross-Country Investigation, Labour Economics, 19, 682–694.
36. Miller, E. 2008. An Assessment of CES and Cobb-Douglas Production Functions: Working Paper 2008-05, Working Papers 19992, Congressional Budget Office. https://www.cbo.gov/publication/19992.
37. Muck, J., P. McAdam, and J. Growiec. 2015. Will the True Labor Share Stand Up? Working paper, 1806, European Central Bank.
38. Muck, J. 2017. Elasticity of substitution between labor and capital: robust evidence from developed economies, NBP Working Papers 271, Narodowy Bank Polski, Economic Research Department: https://www.researchgate.net/publication/331160173_Elasticity_of_substitution_between_labor_and_capital_robust_evidence_from_developed_economies.

39. Meyer, Robert A., and K. R. Kadiyala. 1974. Linear and nonlinear estimation of production functions. *Southern Economic Journal* 40: 463–72.
40. Michetti, Elisabetta. 2013. Complex attractors and basins in a growth model with nonconcave production function and logistic population growth rate. *Mathematics and Computers in Simulation* 108, Doi: 10.1016/j.mat.com.2013.09.001.
41. Nguyen T.N., Kosheleva O., Kreinovich V.: Maximum Entropy Beyond Selecting Probability Distributions. In: Anh L., Dong L., Kreinovich V., Thach N. (eds) Econometrics for Financial Applications. ECONVN 2018. *Studies in Computational Intelligence*, vol 760. Springer, Cham, https://doi.org/10.1007/978-3-319-73150-6_15 (2018)
42. Nguyen T.N., Kosheleva O., Kreinovich V., Nguyen H.P.: Blockchains Beyond Bitcoin: Towards Optimal Level of Decentralization in Storing Financial Data. In: Kreinovich V., Thach N., Trung N., Van Thanh D. (eds) Beyond Traditional Probabilistic Methods in Economics. ECONVN 2019. *Studies in Computational Intelligence*, vol 809. Springer, Cham, DOI: https://doi.org/10.1007/978-3-030-04200-4_12 (2019)
43. Palivos, Theodore, and Giannis Karagiannis. 2004. The Elasticity of Substitution in Convex Models of Endogenous Growth. Unpublished Manuscript.
44. Paul, Saumik. 2019. *Labour Income Share Dynamics with Variable Elasticity of Substitution*. Discussion Paper Series, IZA DP No. 12418. http://ftp.iza.org/dp12418.pdf
45. Pereira, C. 2003. Empirical Essays on the Elasticity of Substitution, Technical Change, and Economic Growth. Ph.D. dissertation, North Carolina State University.
46. Pinheiro, José, and Douglas Bates. 2000. *Mixed-Effects Models in S and S-PLUS*. New York: Springer.
47. Pham, Le Thong, and Phuong Thuy Ly. 2016. Technical efficiency of Vietnamese manufacturing enterprises. *Journal of Economics and Development* 229: 43–51.
48. Revankar, Nagesh S. 1971a. A class of variable elasticity of substitution production functions. *Econometrica* 39: 61–71.
49. Revankar, Nagesh S. 1971b. Capital-labour substitution, technological change, and economic growth: The U.S. experience, 1929–1953, *Metroeconomica* 23: 154–76.
50. Romer, Paul M., 1986. Increasing Returns and Long-Run Growth, *Journal of Political Economy*, 94, 5, pp. 1002–1037.
51. Romer, Paul M., 1987. Crazy Explanations for the Productivity Slowdown, *NBER Macroeconomics Annual*, Cambridge: MIT Press.
52. Roskamp, Karl W. 1977. Labour productivity and the elasticity of factor substitution in West Germany industries. *Review of Economics and Statistics* 59: 366–71.
53. Sato, Ryuzo, and Hoffman, Ronald F. 1968. Production functions with variable elasticity of substitution: Some analysis and testing. *Review of Economics and Statistics* 50: 453–60.
54. Sinelnikov-Murylev, Sergey, Drobyshevsky, Sergey, Kazakova, Maria, and Alexeev, Michael. 2015. *Decomposition of Russian GDP Growth Rates*. Moscow: Gaidar Institute.
55. Sriboonchitta, Songsak, Hung T. Nguyen, Olga Kosheleva, Vladik Kreinovich, and Thach Ngoc Nguyen. 2019. Quantum Approach Explains the Need for Expert Knowledge: On the Example of Econometrics. In *Structural Changes and Their Econometric Modeling, TES 2019, Studies in Computational Intelligence*. Edited by Vladik Kreinovich, Songsak Sriboonchitta. Cham: Springer, vol. 808. doi:10.1007/978-3-030-04263-9_15.
56. Starbuck, William H. 2006. *The Production of Knowledge. The Challenge of Social Science Research*. New York: Oxford University Press.
57. Svítek, Miroslav, Olga Kosheleva, Vladik Kreinovich, and Thach Ngoc Nguyen. 2019. Why Quantum (Wave Probability) Models Are a Good Description of Many Non-quantum Complex Systems, and How to Go Beyond Quantum Models. *In Beyond Traditional Probabilistic Methods in Economics, ECONVN 2019, Studies in Computational Intelligence*. Edited by Kreinovich V., N. Thach, N. Trung and Van Thanh D. Cham: Springer, vol. 809. doi:10.1007/978-3-030-04200-4_13.
58. Thach, Nguyen Ngoc. 2020a. Macroeconomic Growth in Vietnam Transitioned to Market: an Unrestricted VES Framework. *Economies* 8(3): 58. doi: 10.3390/economies8030058.
59. Thach, Nguyen Ngoc. 2020b. How to Explain when the ES is Lower than One? A Bayesian Nonlinear Mixed-effects Approach. *Journal of Risk and Financial Management* 13: 21. doi:10.3390/jrfm13020021.
60. Thach, Nguyen Ngoc. 2020c. Endogenous Economic Growth: The Arrow-Romer Theory and a Test on Vietnamese Economy. *WSEAS Transactions on Business and Economics* 17: 374–86. doi:10.37394/23207.2020.17.37
61. Thach, Nguyen Ngoc, Anh Le Hoang, and An Pham Thi Ha. 2019. The Effects of Public Expenditure on Economic Growth in Asia Countries: A Bayesian Model Averaging Approach. *Asian Journal of Economics and Banking* 3: 126–49.

62. Thach, Nguyen Ngoc, Vladik Kreinovich, and Nguyen Duc Trung, eds. 2020. Data Science for Financial Econometrics. Cham: Springer.
63. Tsang, Herbert H. 1976. A generalized model for the CES-VES family of production function. *Metroeconomica* 28: 107–18.
64. United Nations Development Programme (2020). Human Development Report. http://www.hdr.undp.org/
65. Zellner, Arnold, and Hang Ryu. 1998. Alternative functional forms for production, cost and returns to scale functions. *Journal of Applied Econometrics* 13: 101–27.